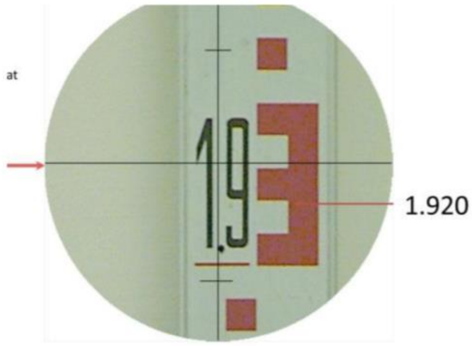
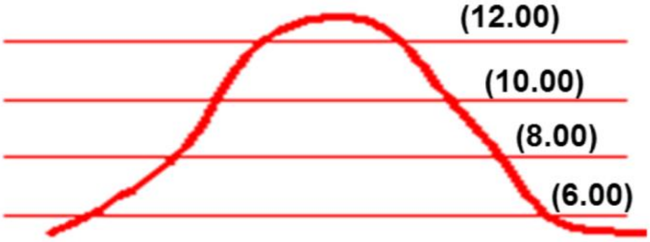
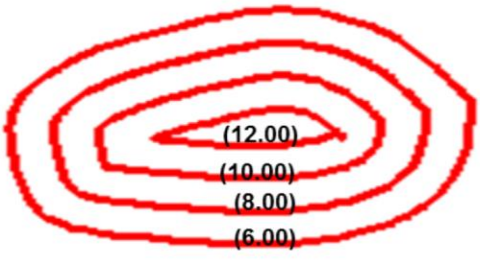
Engineering Surveying

PART I







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بسم الله الرحمن الرحيم

"سبحانك لا علم لنا إلا ما علمتنا"

صدق الله العظيم

PREFACE

This book is designated to cover an important course in surveying for civil engineering students in universities, polytechnics and colleges of technology.

The book prepares the student to perform the duties of any member of surveying party both in the field and in the office, also it serves as a firm foundation for future studies in surveying.

The material within the book has been taken from so many sources that individual acknowledgement is impossible ,however our special thanks are due to all staff members of civil engineering at Beirut Arab University.

The authors

June 2013

Definition of surveying:

Surveying is the measurement of dimensional relationships among points, lines, and physical features on or near the Earth's surface. Basically, surveying determines horizontal distances, elevation differences, directions, and angles. These basic determinations are applied further to the computation of areas and volumes and to the establishment of locations with respect to some coordinate system.

Surveying is typically used to locate and measure property lines; to lay out buildings, bridges, channels, highways, sewers, and pipelines for construction; to locate stations for launching and tracking satellites; and to obtain topographic information for mapping and charting.

Horizontal distances are usually assumed to be parallel to a common plane. Each measurement has both length and direction. Length is expressed in feet or in meters. Direction is expressed as a bearing of the azimuthal angle relationship to a reference meridian, which is the north-south direction. It can be the true meridian, a grid meridian, or some other assumed meridian.

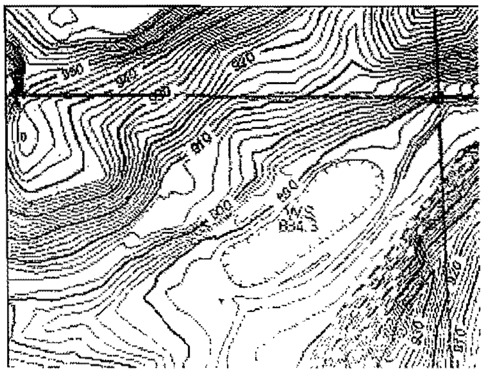
Reference, or control, is a concept that applies to the positions of lines as well as to their directions. In its simplest form, the position control is an identifiable or understood point of origin for the lines of a survey.

Vertical measurement adds the third dimension to an object's position. This dimension is expressed as the distance above some reference surface, usually mean sea level, called a datum. Mean sea level is determined by averaging high and low tides during a lunar month.

Types of surveys:

Engineering surveys: those surveys associated with the engineering design (topographic, layout and as-built) often. And this is the most important type of survey for construction to determine the necessary areas and volumes of land and materials that may be required during construction. Also to produce up-to-date plans of the areas in which Engineering projects are to be built.

Topographic survey: a survey that measures the elevation of points on a particular piece of land, and presents them as contour lines on a plot.



Geodetic survey: Geodetic surveys cover such a large area that curved shape of the Earth has to be taken into consideration. Such surveys involve advanced mathematical theory and require precise measurements to provide a framework of accurately located points.

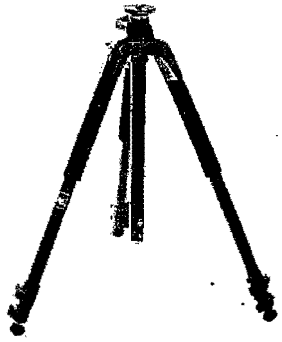
These points can be used to map entire continent, to measure the true size and shape of the Earth.

photogrammetry: Surveys at most scale may be undertaken by photogrammetry , using photographs taken with special cameras from an aircraft or on the ground. Maps or numerical data can be produced , usually with aid of sophisticated and expensive stereo-plotting machines and computers.

Hydrographic survey: a survey conducted with the purpose of mapping the coastline and seabed for navigation, engineering, or resource management purposes.

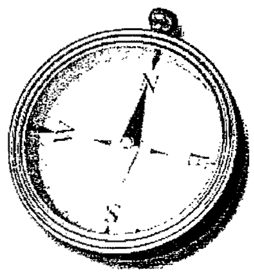
Instruments and Techniques:

A tripod: is a portable three-legged frame, used as a platform for supporting the weight and maintaining the stability of some other object. A tripod provides stability against downward forces and horizontal forces and movements about horizontal axes.

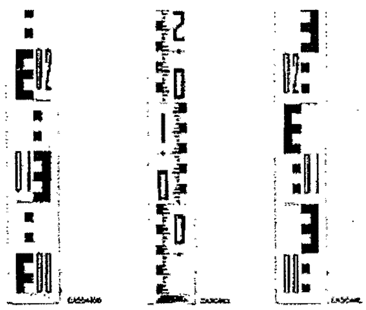


Ranging pole: Surveying instrument consisting of a straight rod painted in bands of alternate red and white each one foot wide; used for sightings by surveyors

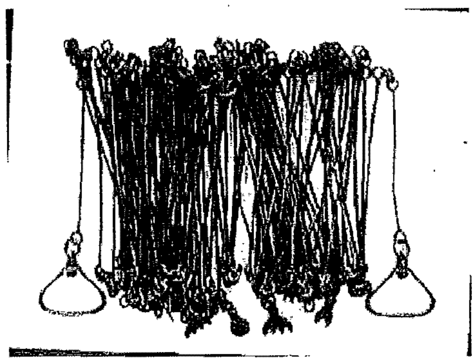
A compass: Is a navigational instrument that shows directions in a frame of reference that is stationary relative to the surface of the earth. The frame of reference defines the four cardinal directions (or *points*) – north, south, east and west.



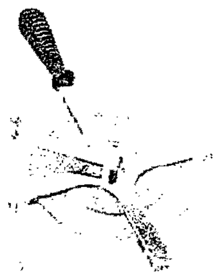
A level staff: Also called leveling rod, is a graduated wooden or aluminum rod, the use of which permits the determination of differences in elevation.



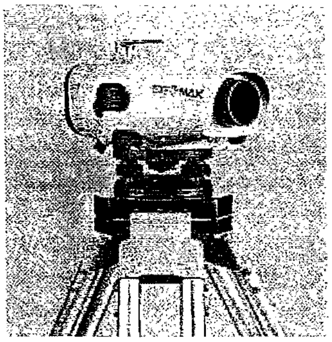
- ***The engineer's chain***: 100 ft (30 m) long and also consisting of 100 links.



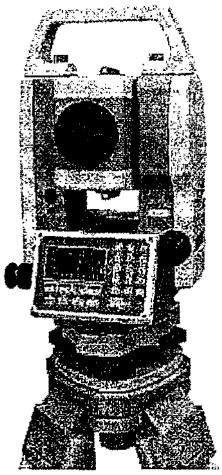
- ***The tape:*** usually of steel, this has largely superseded chains; and the rod. Tapes and rods made of Invar metal (an alloy of steel and nickel) are used for very precise work because of their low coefficient of thermal expansion.



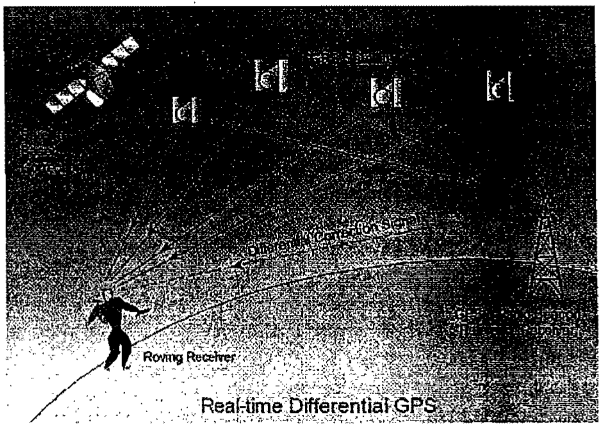
The level: The height of points in relation to a datum line (usually mean sea level) is measured with a leveling instrument consisting of a telescope fitted with a spirit level and usually mounted on a tripod. It is used in conjunction with a leveling rod placed at the point to be measured and sighted through the telescope.



A total station: is an electronic/optical instrument used in modern surveying and building construction. The total station is an electronic theodolite (transit) integrated with an electronic distance meter (EDM) to read slope distances from the instrument to a particular point.
Robotic total stations allow the operator to control the instrument from a distance via remote control. This eliminates the need for an assistant staff member as the operator holds the reflector and controls the total station from the observed point.



GPS: The Global Positioning System (GPS) is a space-based satellite navigation system that provides location and time information in all weather conditions, anywhere on or near the Earth where there is an unobstructed line of sight to four or more GPS satellites. The system provides critical capabilities to military, civil and commercial users around the world. It is maintained by the United States government and is freely accessible to anyone with a GPS receiver.



Scale of Map

Scale of maps

- Scale of map = $\dfrac{\text{The map distance (d)}}{\text{The real ground distance (D)}}$

- **If the scale 1 : 500**
  1 mm on map =500 mm on ground
  2 mm on map =1.00 m on ground

Presentation of the scale

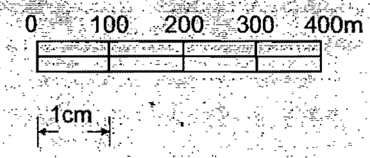
- As a ratio
- Graphical representation
  -Linear scale
  -Grid or diagonal scale

scale

- Scale of drawn map with two forms :

Numerical
1:5000 , 1:25000 , 1:50000 ,........

Graphical

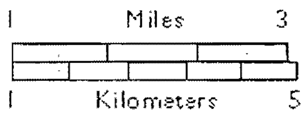


scale 1:10000

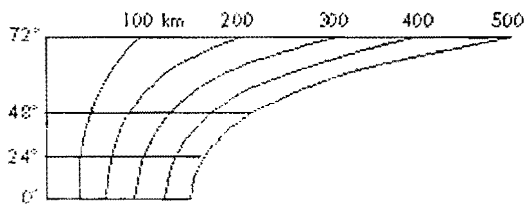
scale

- Scale of drawn map with two forms :

Graphical



Bar Scale



Variable Scale

Types of maps scales

- Large scale 1:10 ⟶ 1:2500
- Used in site planes and in planning and designing of civil engineering works such as streets,roads,water network and sewage.

Types of maps scales

- Medium scales 1:5000 ⟶1:25 000
  - Used in town maps, highways and railways maps
- Small scales 1:25 000 ⟶ 1:2000 000
  - Used in geographic maps because it is cover a huge areas

Question (1) :

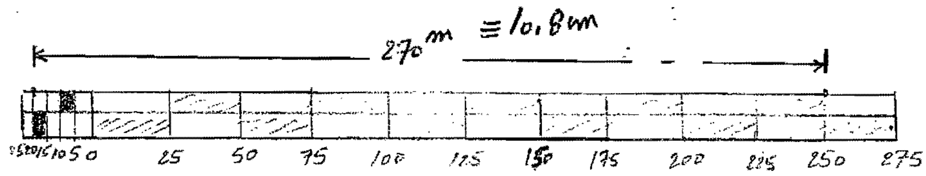
a- Calculate the scale of plan where 1cm represents 24.45 m.

$\text{Scale} = \dfrac{\text{Dimension on map}}{\text{Dimension on site}} = \dfrac{1\ cm}{24.45\ m} = \dfrac{1\ cm}{2445\ cm} = \dfrac{1}{2445}$, say $\boxed{\dfrac{1}{2500}}$

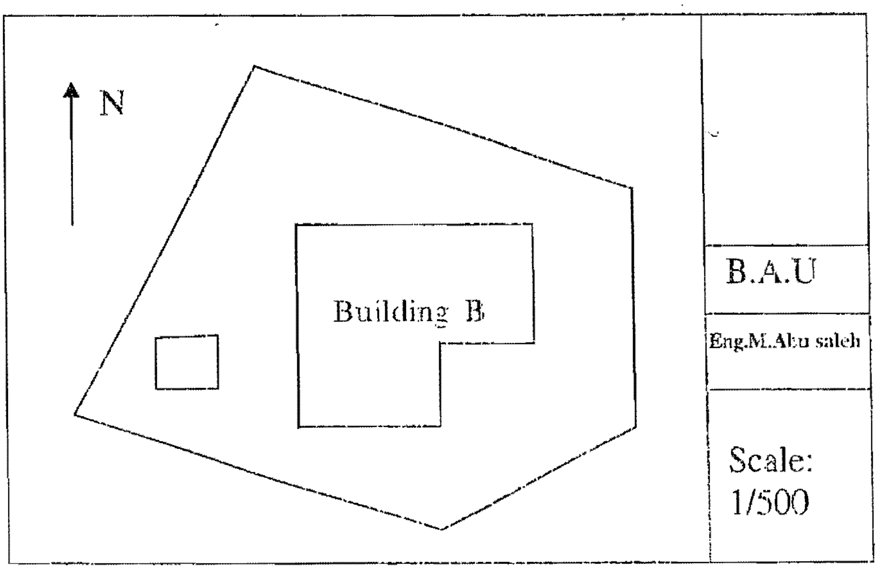
b- A land area was investigated **150cm^2** on map if a scale of **1/1000** was used Determine the actual area of the land in hectares.

$A_{actual} = (150) \times 10^6 = 15000\ m^2 = 1.5\ ha$

c- Draw a scale for a map scaled by 1:2500 to read 10m, and find out the length 270m.

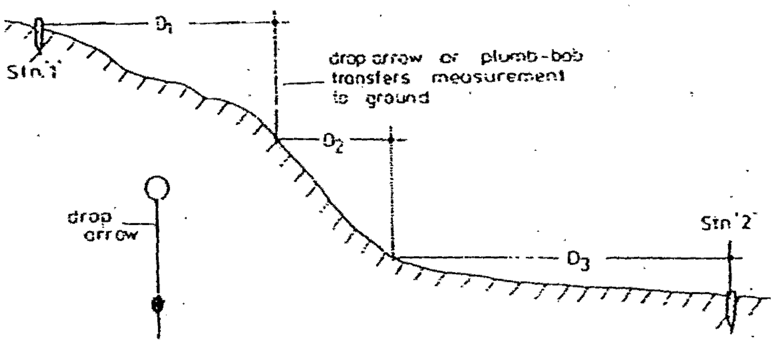


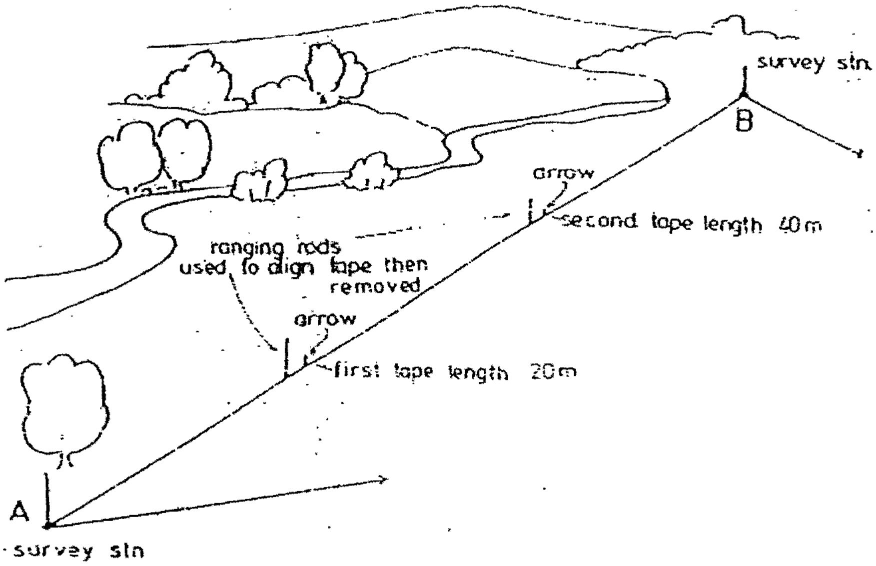
— Calculate the true area (on site) of Building (B) shown on the following map:

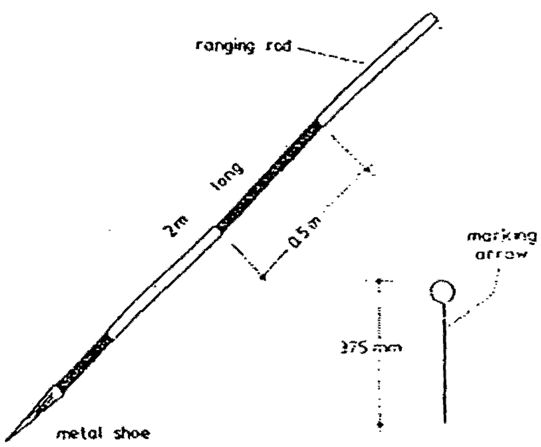


Linear Surveying

Tape Measurement of Lines



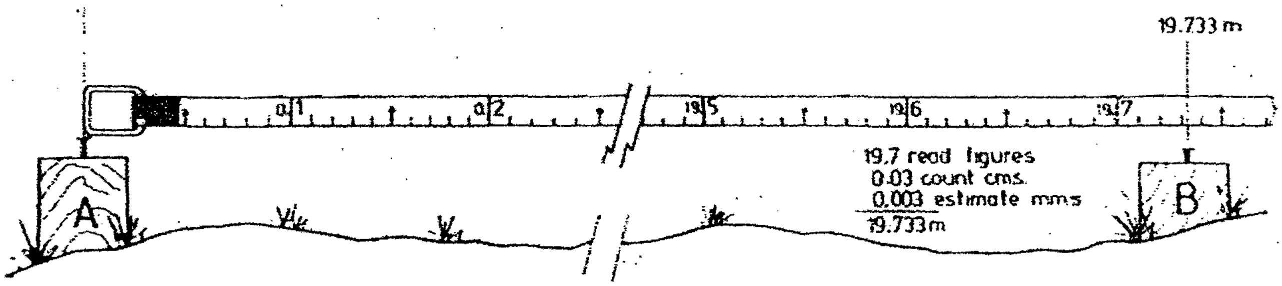




Tape Measurement of Lines

On every survey, there will inevitably be a variety of long and short, flat and inclined, lines to be measured accurately.

A) The figure below shows a short survey line AB marked on the ground by two pegs. The distance AB is shorter than one tape length.

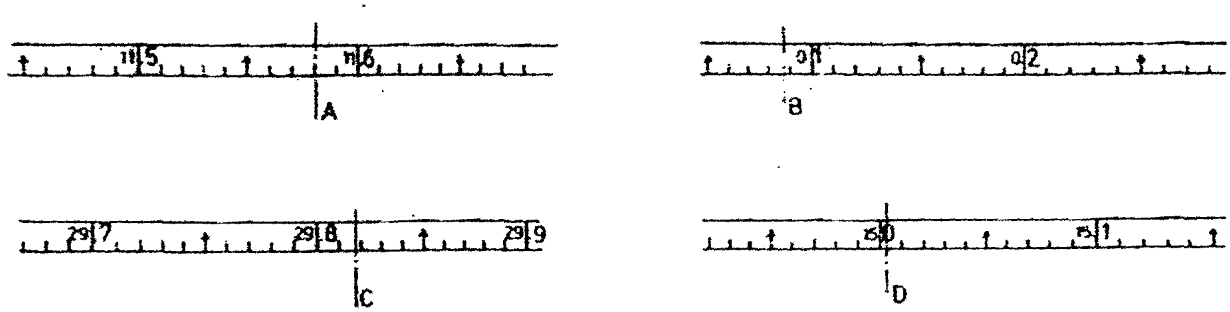


Measurement of a short line is obtained by the following steps:

1- Unreel the tape and straightening it along the line between the pegs.

2- The zero point of the tape is held against station A by the rear tape person (called the follower).

3- The forward end of the tape is read against station B by the forward tape person (called the leader) after it has been carefully tightened.

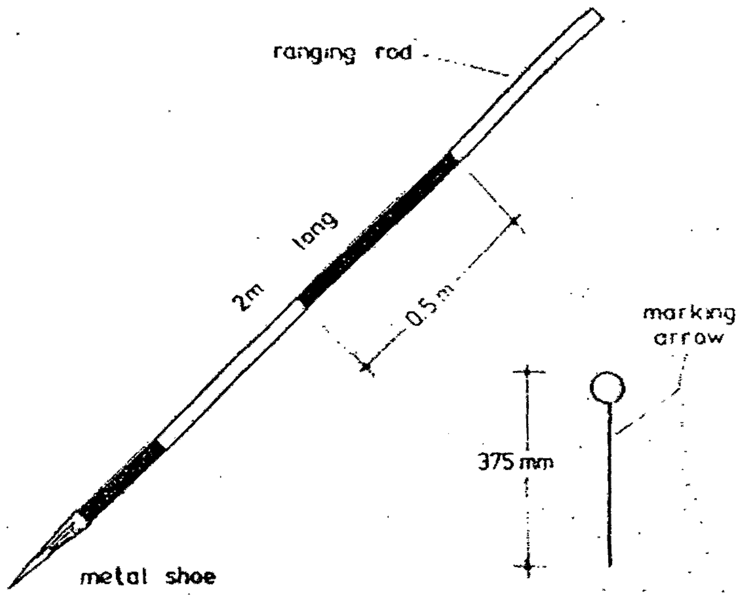


B) On surveys most of the lines will be considerably longer than one tape length and a sound operational technique is required.

- Two ancillary pieces of equipment are necessary:

i- Ranging rods.

ii- Marking arrows.



Measurement of a long line is obtained by the following steps:

1- The follower holds the zero end of the tape against station A and the leader pulls the tape towards station B.

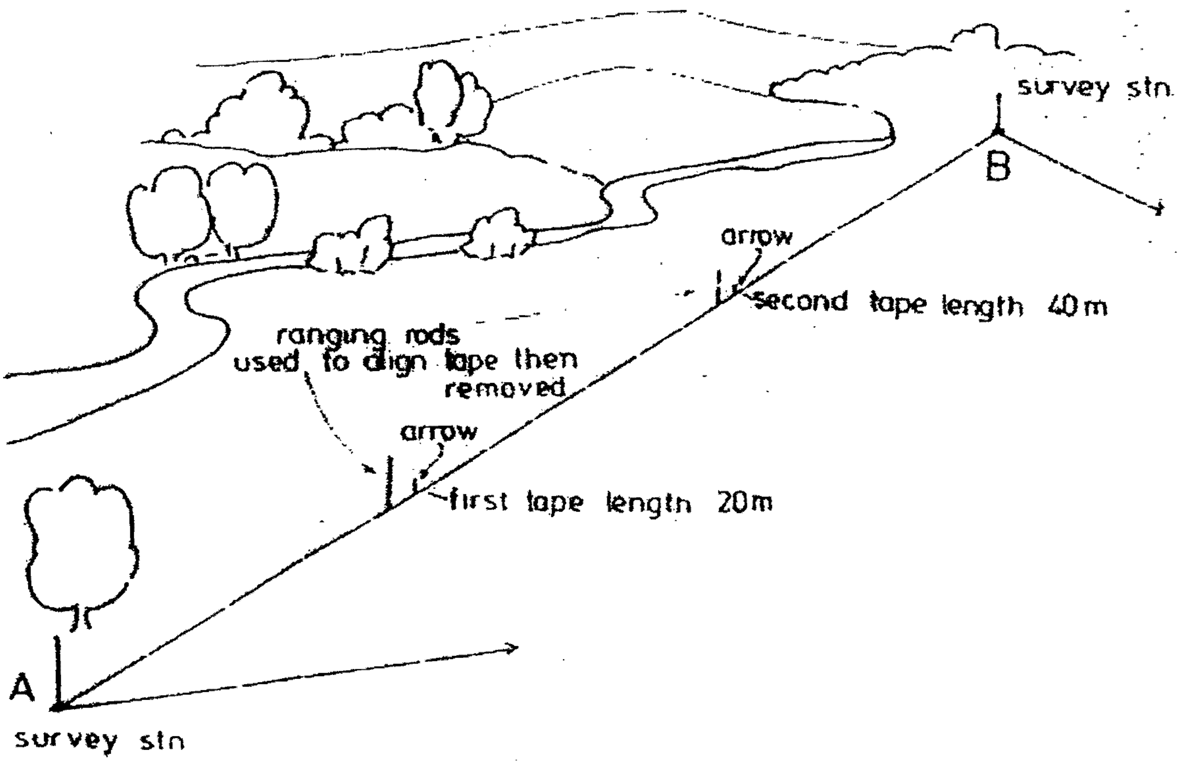
2- When the tape has been laid out the leader holds the ranging rod vertically approximately on the line.

3- The follower signals to the leader to move it until it is exactly on the line AB.

4- The tape is tightened between the newly erected rod and station A and an arrow is pushed into the ground at the 20 mark of the tape.

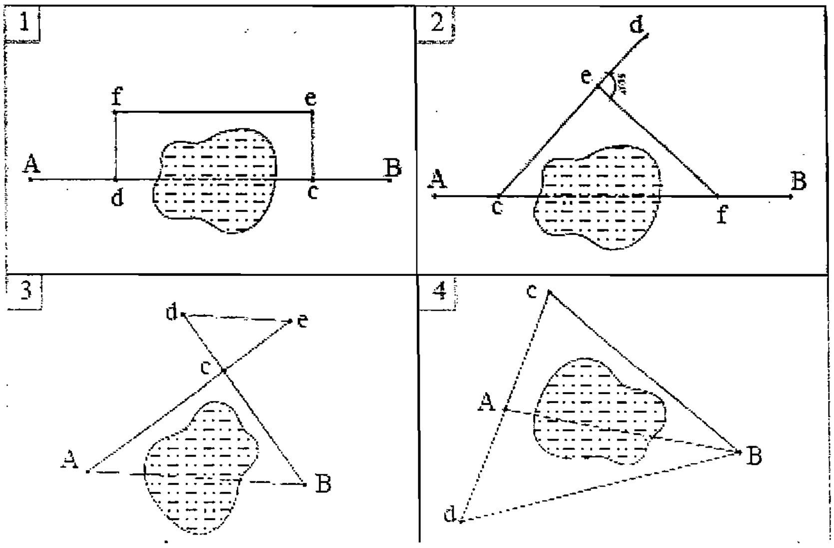
5- The follower moves forward to this new point and the whole procedure is repeated for the remainder of the line until station B is reached.

6- The follower gathers the marking arrows and the number of tape lengths measured is the number of the arrows carried by the follower, the portion of tape between the last arrow and station B is then measured and added to the number of complete tape lengths to produce the total length of the line.



Obstacles:

Some way , chain lines should not be broken by obstacles, it often happens that an unhindered line is not attainable. In the following figure we show some solutions of obstacles:



Correction of linear measurements

1-tape correction:

$$L_c = L_m \cdot (l'/l)$$

L_c : the required corrected length of line (true length)

L_m : the measured length of line

l' : the actual length of tape (longer or shorter than the standard)

l : standard length of tape (nominal length 20 m , 30 m ,50m ...)

Note: for Areas calculation the rule will be:

$$A_c = A_m \cdot (l'/l)^2$$

A_c: correct area (true area)

A_m: measured area

2-Slope correction:

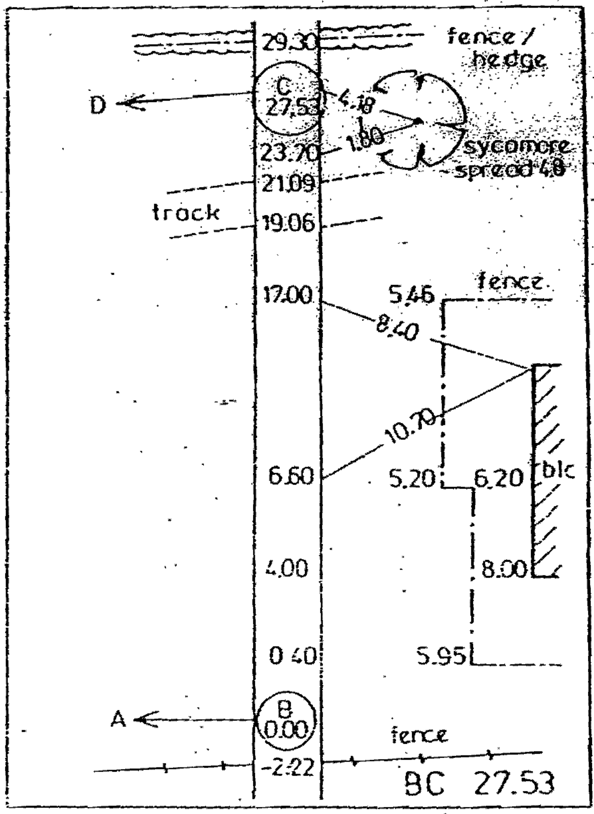
$$D = L \cdot \cos\alpha$$

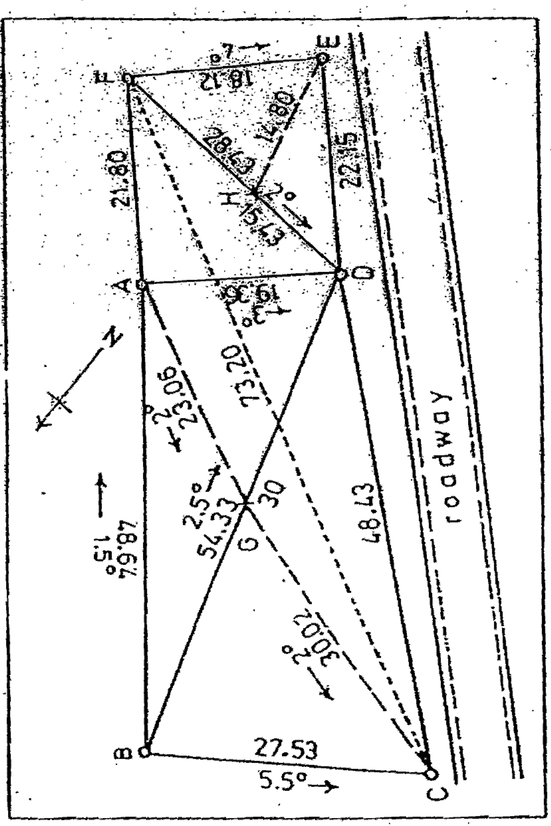
D: the required horizontal distance (plan length)

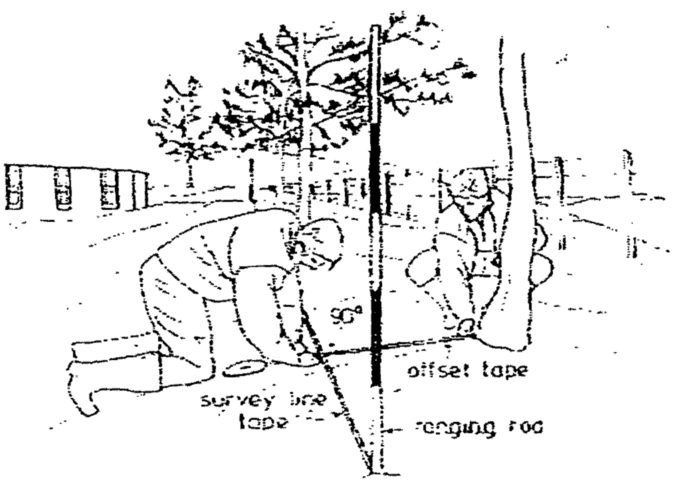
L: inclined length and α: angle of inclination (slope)

Note: if the difference of height is given use $D = \sqrt{L^2 - h^2}$

Linear Surveying







Linear Surveying

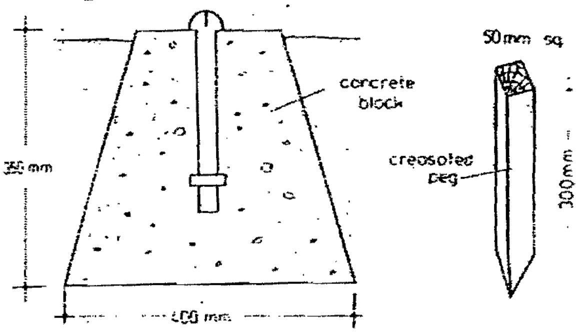
Where comparatively small areas have to be surveyed a linear survey might be used.

The principle of linear surveying is to divide the area into a number of triangles, all the sides of which are measured.

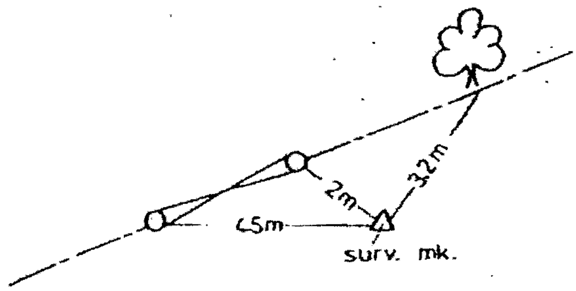
Linear surveying of a site is obtained by the following steps:

1- Draw a sketch of the site and locate the north direction on it.

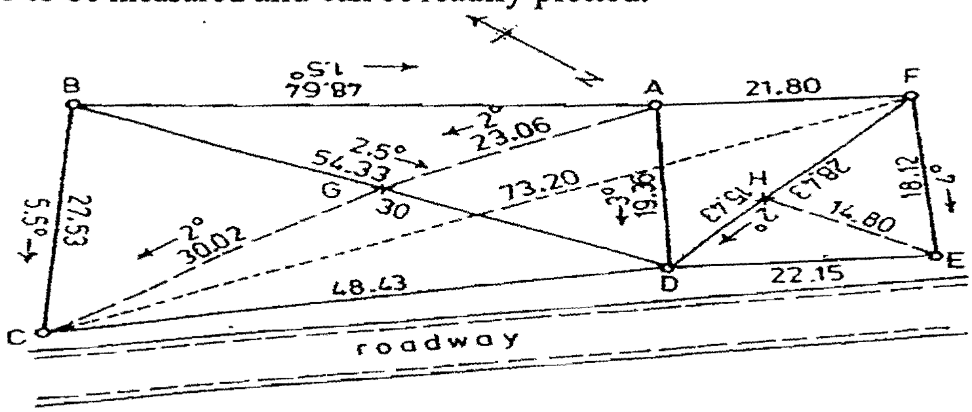
2- Select the traverse station points and mark them with a suitable mark.



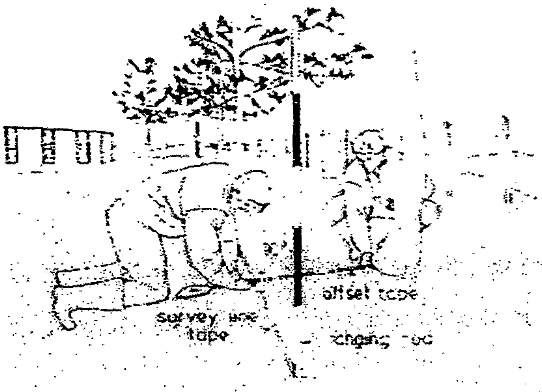
3- Reference the traverse station points to be able to locate them again easily.



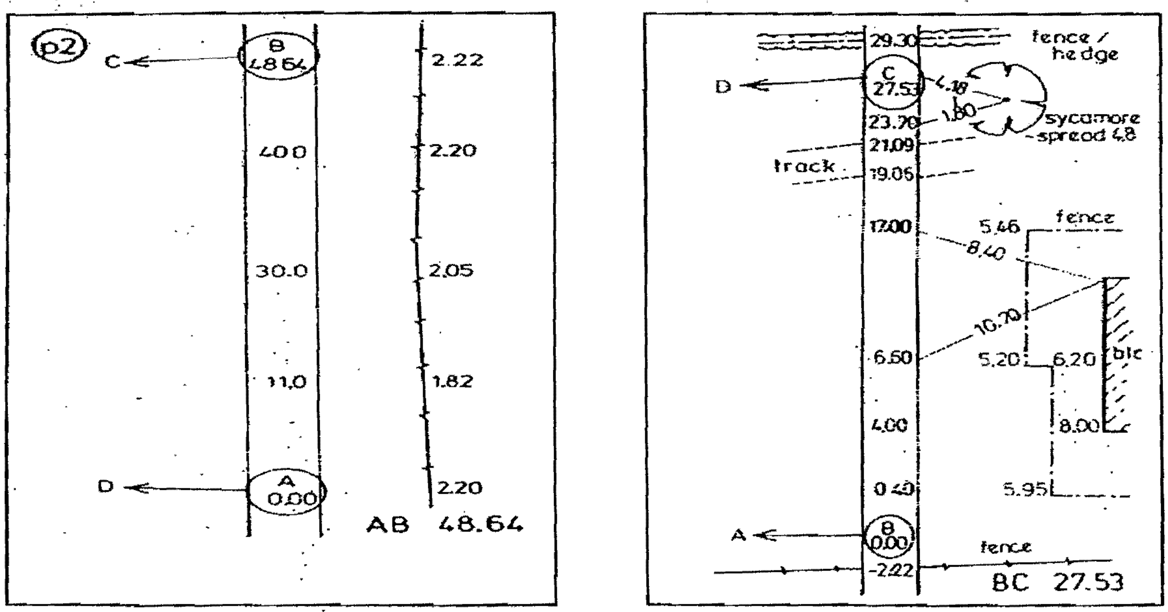
4- Establish a framework over the whole site to form a geometric figure, which all sides are to be measured and can be readily plotted.



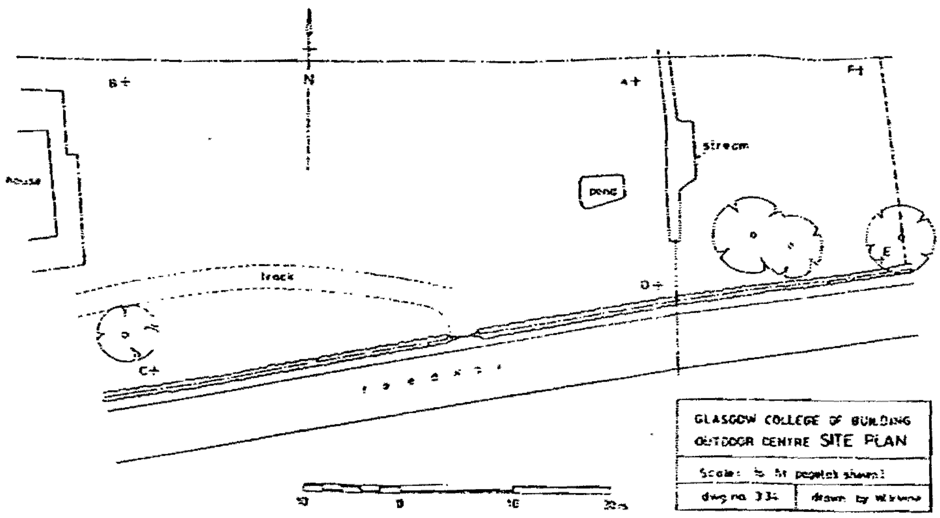
5- All the details (fences, walls, streams, trees, buildings, etc.) are then added to the framework by offsetting.



6- Record the surveying details for each line of the framework on the survey field book as shown.

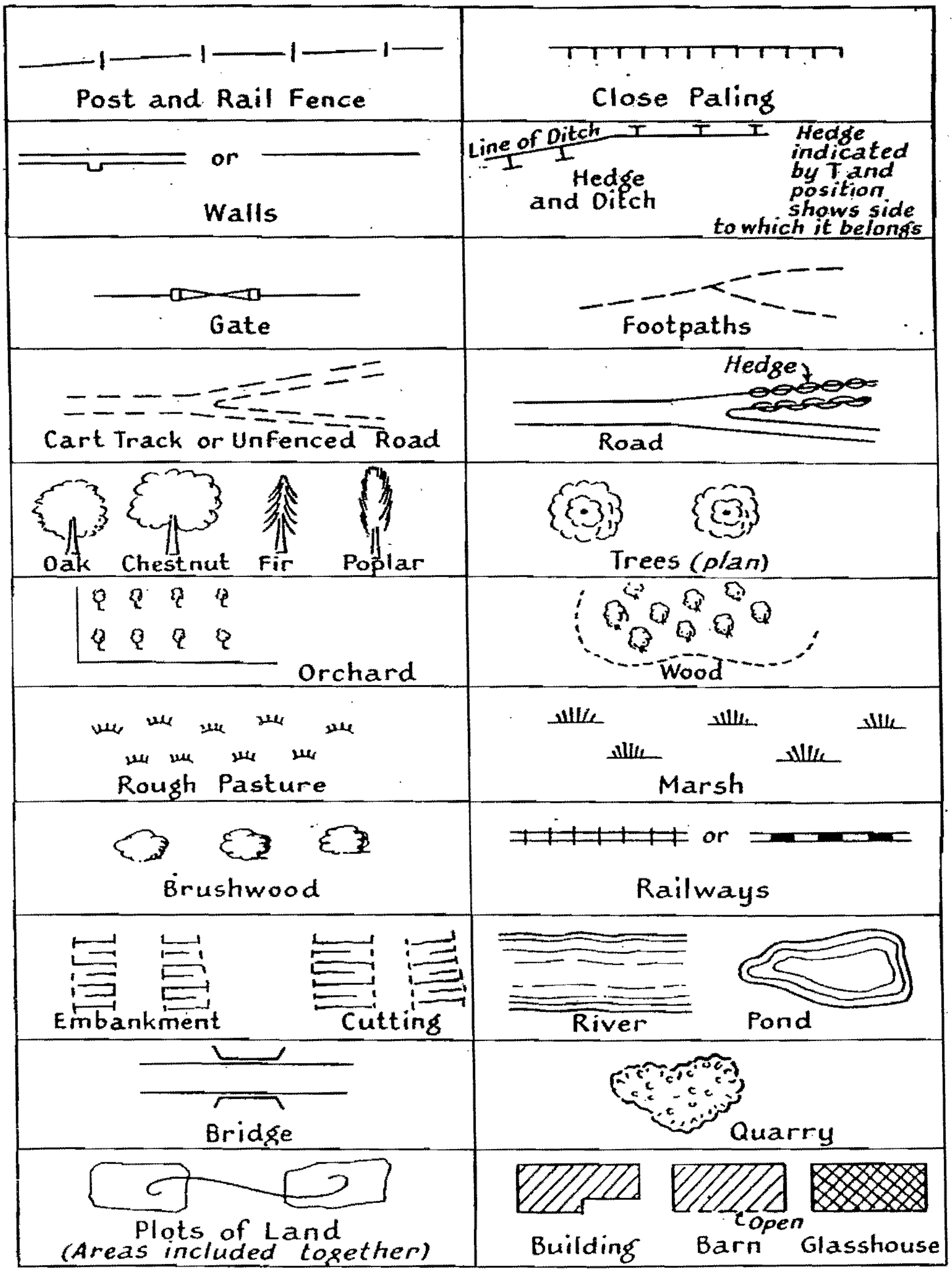


7- Finally the survey is to be plotted starting with the framework followed by the details.



LEGEND

Some Conventional Signs:



- Calculate the corrected plan length of line AC measured in 2 sections as follows :

AB= 152.85ms. angle of slope= $5^\circ 30'$
BC= 48.50 ms. Difference in elevation between B & C=2. 5 m
Where steel tape which was actually 19.9 ms.

--<u>tape correction</u>:---AB'=152.85x(19.9/20)= 152.085 m--------

------------------------BC'=48.25m

--<u>slope correction</u>:---AB_{plan} = 152.085xcos($5^\circ 30'$)= 151.385 m------

-------------------------BC_{plan} = 48.435 m ----------------------------

-------AC_{plan} =AB_{plan}+ BC_{plan} =199.82 m ----------------------

- Fig.(1) shows a survey framework A-F. Add a series of suitable check lines to the survey.

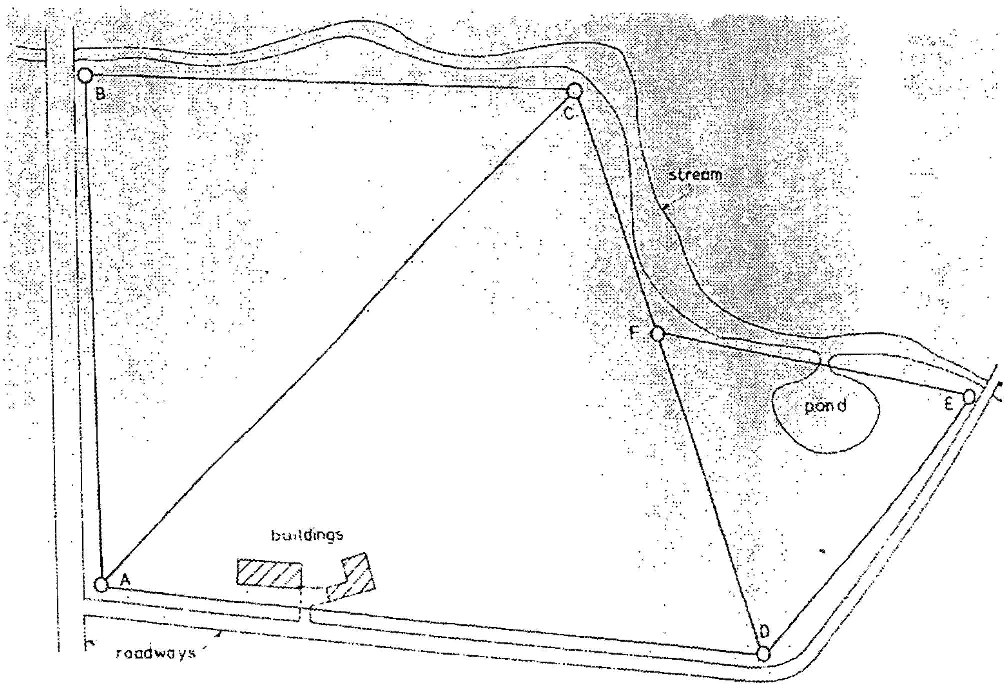


Figure (1)

- line **AD** was measured in three sections as follows :

AB= **102.50 m** angle of slope= **$6^{\circ}30'$**

BC= **57.95 m** angle of slope= **$2^{\circ}00'$**

CD=**48.20 m** angle of slope= **$-5^{\circ}00'$**

If the tape used was later found **10 cm** too long than the standard **30m.**

Determine: a) the corrected plan length of line AD;

b) the length of AD on a map drawn by **scale 1/2500**

c) the difference in elevation between A and D;

d) the gradient of a road to be made between A and D

Solution:

a) Tape correction: $L_c = L_m . (l'/l)$

$AB_c=102.50(30.1/30)=102.84$ m
$BC_c=57.95(30.1/30)=58.14$ m
$CD_c=48.20(30.1/30)=48.36$ m

Slope correction: $D = L . \cos\alpha$

$AB_{plan}=102.84\cos(6^{\circ}30')=102.18$ m
$BC_{plan}=58.14\cos(2^{\circ}00')=58.10$ m
$CD_{plan}=48.36\cos(-5^{\circ}00')=48.17$ m

So the final corrected plan length of AD is:

$AD_{plan}= AB_{plan}+ BC_{plan}+ CD_{plan}=$**208.45 m**

b) on Map: scale (1/2500)

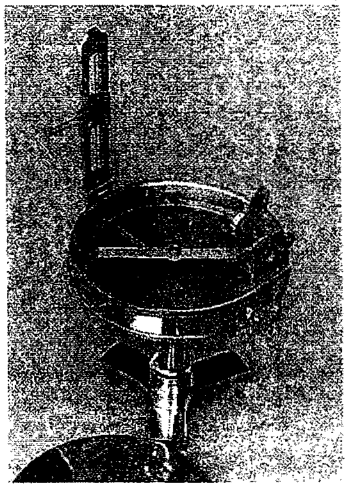
$AD_{on\ map}=(208.45\times100)/2500=$**8.34 cm**

c) $H_{between\ A\&D}=102.18\tan(6^{\circ}30')+58.10\tan(2^{\circ}00')+48.17\tan(-5^{\circ}00')=$9.45m

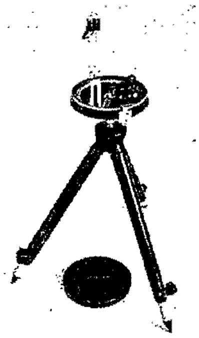
d) gradient of Road AD= $H_{between\ A\&D}/AD_{plan}=$**4.53%**

Compass Surveying

Prismatic compass



- The prismatic compass is used to determine the north direction which is the Magnetic north not the true north.

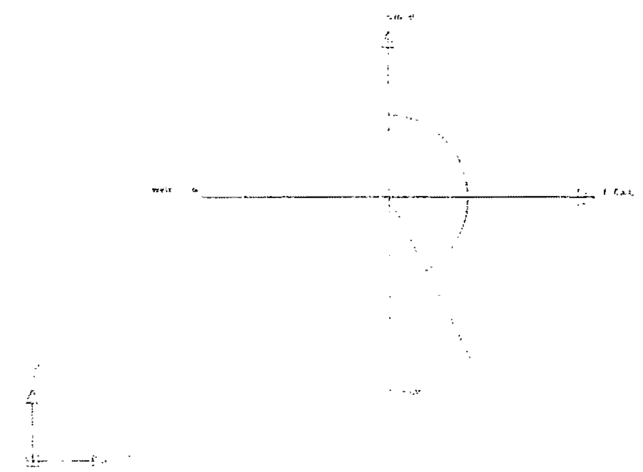


- In general the aim of compass is to determine the Magnetic bearing of certain line.

What is the Bearing of line?

- The bearing of line is the angle From north direction to this line in clock wise direction.
- if it is measured by compass is called Magnetic Bearing.(variable due the time and the location)

- if it is measured Geographically from geographic north is the true Bearing (constant for all the time).

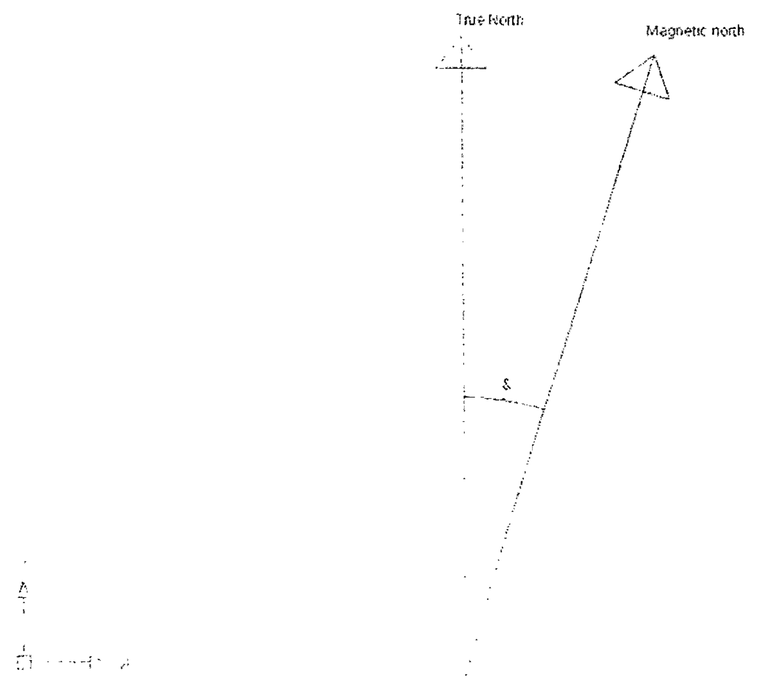


What is the declination?

- The declination is the angle between the Magnetic north and true north (East or West from true North).
- <u>Example:</u>

D = 3°E (i.e. the magnetic north is at 3 degree East of True North).

D= 2°W (i.e. the magnetic north is at 2 degree West of True North).



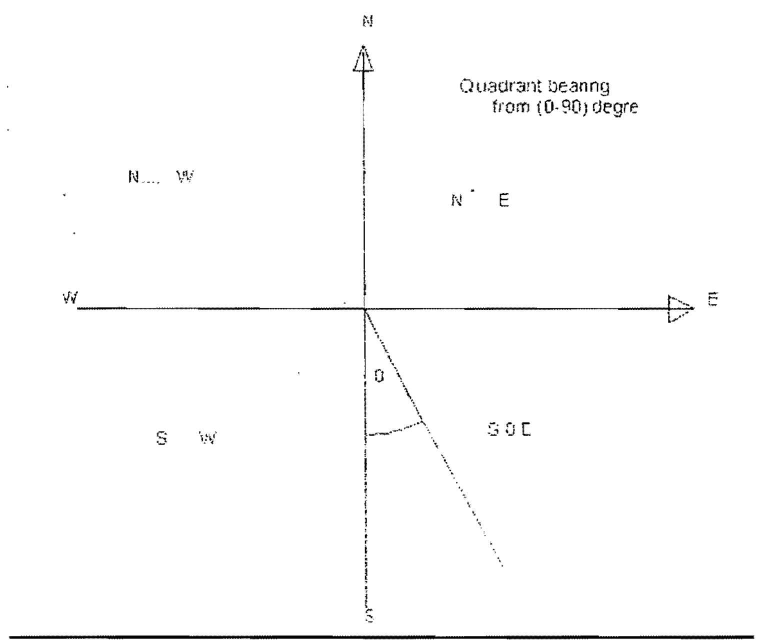
Forward and Back Bearing

_if we have a line [AB] ,the forward Bearing of this line is the Bearing of vector AB(ranging from A to B) But the Back Bearing of AB(B.B.of AB) is the Bearing of Vector BA (ranging from B to A).

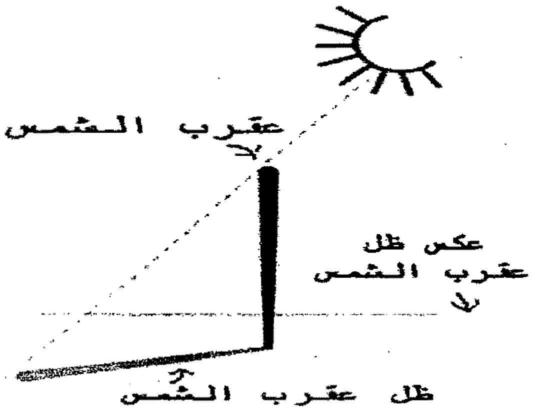
Whole circle Bearing and quadrant Bearing

The whole circle bearing (WCB): is the angle in clock wise direction from north (magnetic or true) to the line from (0-360) degree.

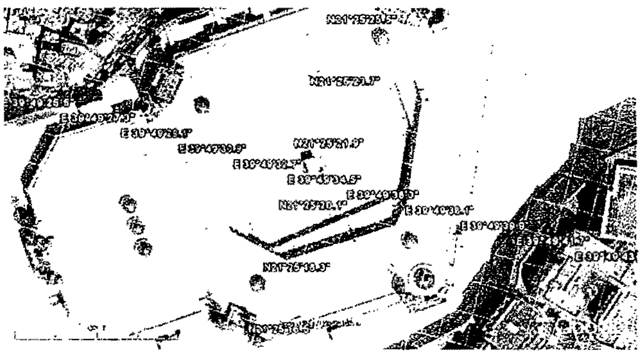
The quadrant Bearing(QB) or Reduced Bearing (RB): is the angle with the vertical Direction (south or North) from 0-90 degree with precision of Quadrant.



عند الظهر يكون ظل عقرب الشمس في اتجاه الشمال الجغرافي



أثبتت الأبحاث العلمية أن الكعبة المشرفة هي مركز الثقل المغناطيسي
وفيها يتطابق الشمال المغناطيسي مع الشمال الجغرافي



Example 1

The magnetic bearing of line *AB* was recorded as S43°-30′E in 1888. If the declination was 2°-00′E, what is the true bearing of the line?

Solution: A sketch is made (Fig. 2) in which the true directions (N, S, E, W) are shown as solid lines and the magnetic directions are shown as dotted lines. Magnetic north is shown 2°-00′E or clockwise of true north and line AB is shown in its proper position 43°-30′E of magnetic south. Once the sketch is completed, the true bearing of the line is obvious.

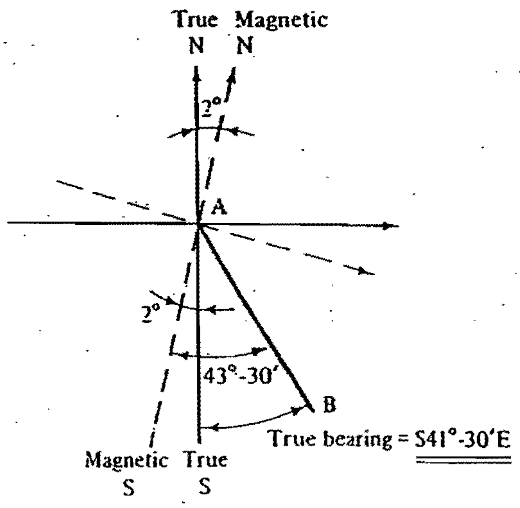


FIGURE 2

Arbitrary north

It may not be necessary to obtain absolute reference and often the first leg of the traverse is assumed to be 0° 00′.

Example 2 True north is 0° 37′ E of Grid North (Fig.3). Magnetic declination in June 1962 was 10° 27′ W.

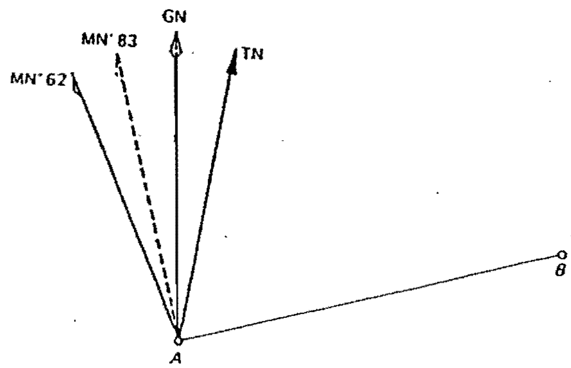


Fig. 3

If the annual variation was 10′ per annum towards North and the grid bearings of line *AB* 082° 32′, what was the magnetic bearing of line *AB* in January 1983?

Grid bearing	082° 32′
Correction	−0° 37′
True bearing	081° 55′
Magnetic declination in June 1962	10° 27′
Magnetic bearing in June 1962	092° 22′
Variation for January 1983 − $20\frac{1}{2} \times 10'$	−3° 25′
Magnetic bearing in January 1983	088° 57′

SOLUTION

Question (1):

Complete the following table:(illustrate your answer with sketch)

Magnetic WCB (forward)	34°	98°	265°	300°
Magnetic declination	10°E	7°W	5°E	11°W
True WCB(forward)	44°	91°	270°	289°
True WCB (back)	224°	271°	90°	109°
True QB (back)	S 44°W	N 89°W	East	S 71°E

Question (2):

In the following table correct the bearings from local attraction errors then calculate the internal angle at A and B after correction.

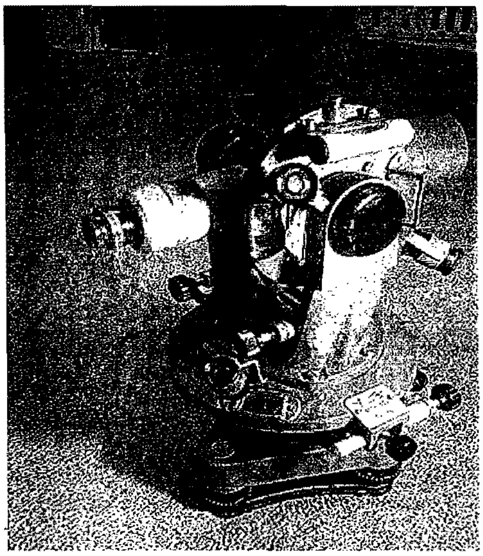
line	F.B	B.B	Difference
AB	042° 00′	222° 10′	180° 10′
BC	105° 30′	284° 40′	179° 10′
CD	209° 00′	028° 50′	180° 10′
DE	267° 50′	086° 10′	181° 40′
EA	316° 10′	135° 50′	180° 20′

line	F.B	B.B	Check
AB	042° 05′	222° 05′	180° 00′
BC	105 05	285 05	180 00
CD	208 55	028 55	180 00
DE	267 00	087 00	180 00
EA	316 00	136 00	180 00

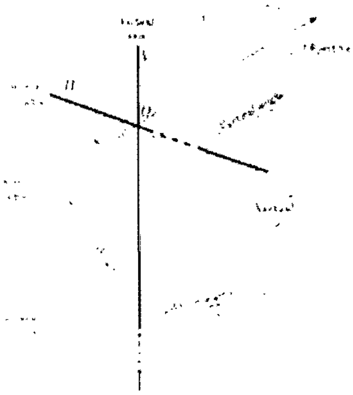
Theodolite Surveying

Theodolite

A theodolite is an instrument for measuring both vertical and horizontal angles. It is considered these days as an old instrument since it is replaced by the new instruments such as total stations. It is important in surveying but it is more adapted of specialized purposes such as metreology and rocket launch technology. A modern theodolite consists of a telescope mounted within two perpendicular axes, the horizontal axis, and the vertical axis. When the telescope is pointed at a desired point, the angle of these axis can be measured with great precision.

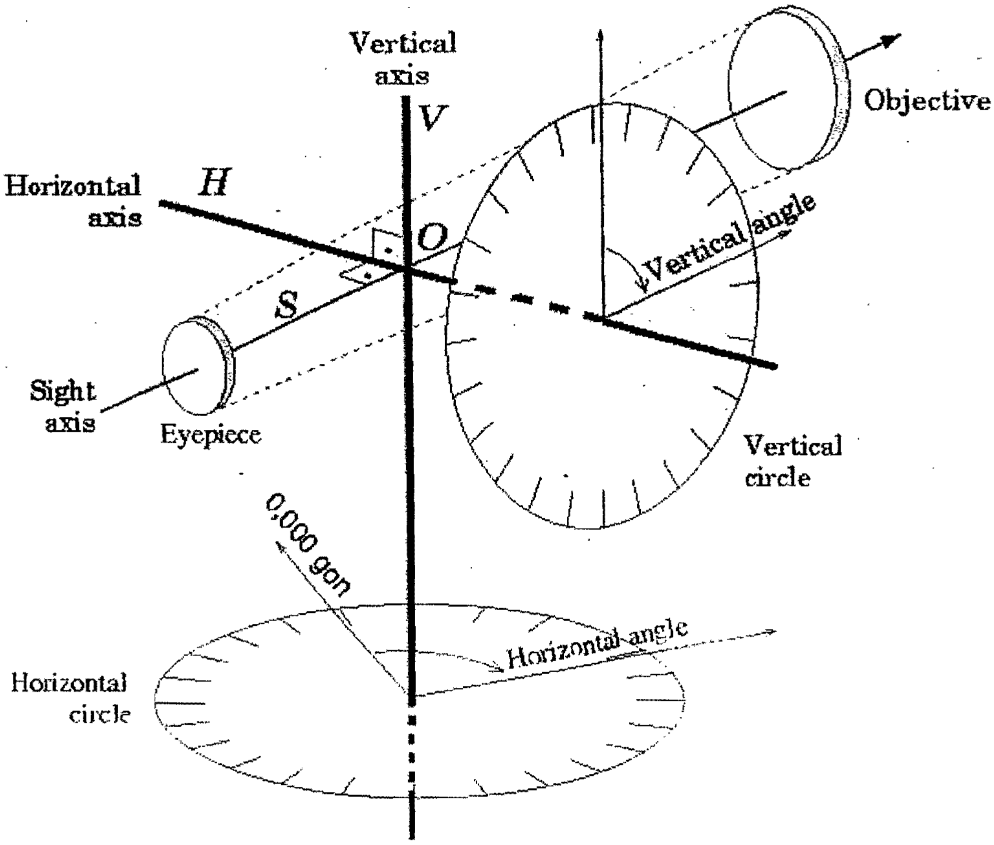


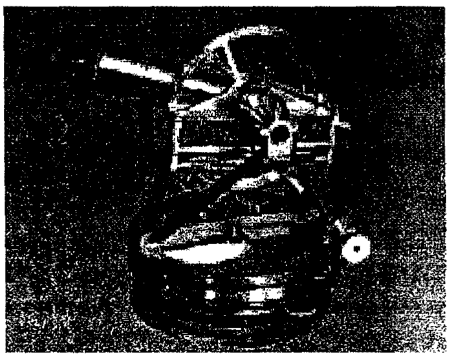
Old primitive Russian Theodolite



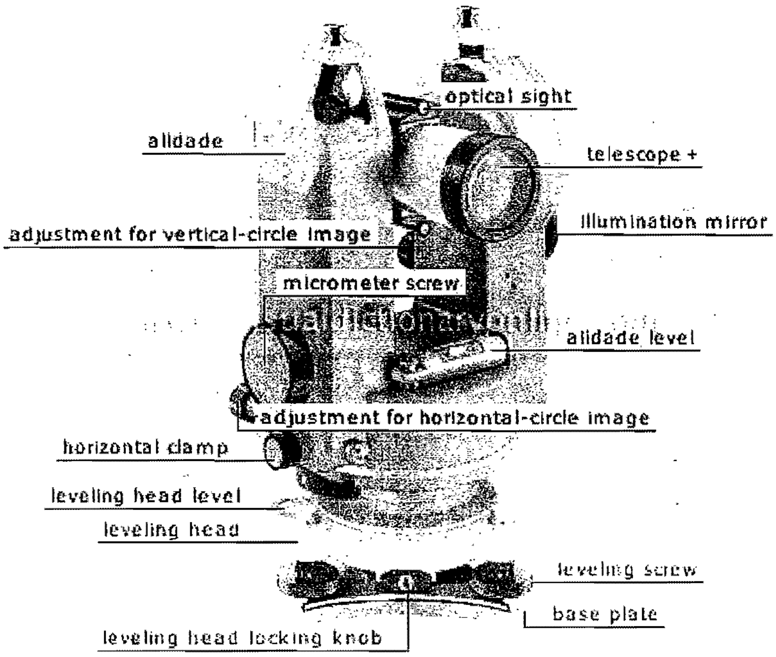
Both axis of a theodolite are equipped with graduated circles that can be read through magnifying lenses. The vertical circle (the one associated with the horizontal axis) should read 90° or 100grad when the sight axis is horizontal (300 grad or 270°). Half of difference with 300 grad is called index error. The horizontal and vertical axis must be perpendicular. When they deviate perpendicularly it is called an horizontal axis error. Other kinds of error also exist in the instrument. All these kinds of error are adjusted through calibration. A theodolite is mounted on the tripod head by means of a forced centering plate or tribrach.

Tacheometer is an instrument for rapid surveying, since it gives both vertical and horizontal measurements of a point on the earth surface. It is considered as a new developed theodolite since it directly gives the distance and angle without hand calculations.

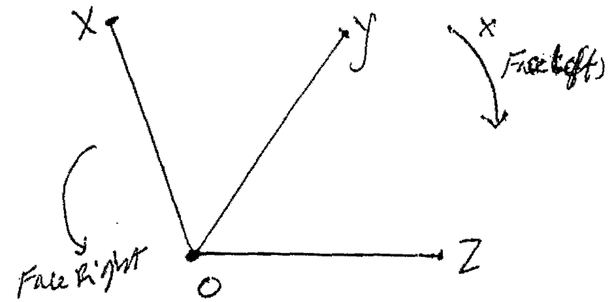




Theodolite in details



occupied stn.	observed stn.	Face Left ° ' "	Face Right ° ' "	Mean ° ' "	Direction ° ' "	Corrected. Dir. ° ' "	Angles ° ' "
O	X	124° 14' 00"	304 13' 10"	124 13 35	00 00 00	00 00 00	
							55° 29' 15"
	Y	179 43 20	359 42' 40"	179 43 00	55 29 25	55 29 15"	
							69 58' 55
	Z	249 42 10	69 42 00	249 42 05	125 28 30	125 28 10"	
							~~23~~4° 31' 5[illegible]
	X	124 14 10	304 14 00	124 14 05	00 00 30"	00 00 00	
							Σ (check)
							360° 00 00
							O.K



$e = +30''$ $-\frac{1}{3}$ Y

$-\frac{2}{3}$ Z

$-\frac{3}{3}$ X

3 points / 3

Traverses

Traverses

Traverse is a method in the field of surveying to establish control networks. It is also used in geodesy. Traverse networks involve placing survey stations along a line or path of travel, and then using the previously surveyed points as a base for observing the next point. Traverse networks have many advantages, including:

- Less reconnaissance and organization needed;
- While in other systems, which may require the survey to be performed along a rigid polygon shape, the traverse can change to any shape and thus can accommodate a great deal of different terrains;
- Only a few observations need to be taken at each station, whereas in other survey networks a great deal of angular and linear observations need to be made and considered;
- Traverse networks are free of the strength of figure considerations that happen in triangular systems;
- Scale error does not add up as the traverse is performed. Azimuth swing errors can also be reduced by increasing the distance between stations.

Types

Frequently in surveying engineering and geodetic science, control points (CP) are setting/observing distance and direction (bearings, angles, azimuths, and elevation). The CP throughout the control network may consist of monuments, benchmarks, vertical control, etc.

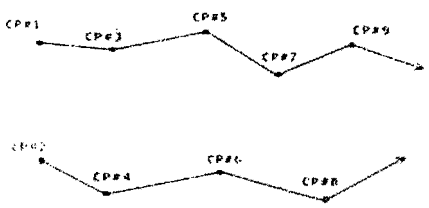


Diagram of an open traverse

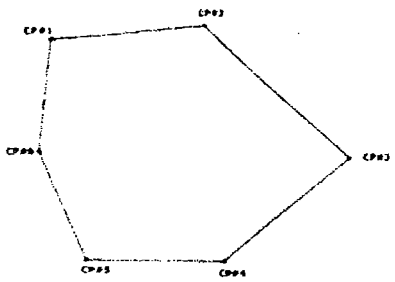


Diagram of a closed traverse

Open/Free

An open, or free traverse (link traverse), consists of a series of linked traverse lines which do not return to the starting point to form a polygon.

- Open survey is utilised in plotting a strip of land which can then be used to plan a route in road construction.

Closed

A closed traverse (polygonal, or loop traverse) is a series of linked traverse lines where the terminal point closes at the starting point.[] A closed traverse enables a check by plotting or computation, with any gap called the linear misclosure. When within acceptable tolerances, the misclosure can be distributed by adjusting the bearings and distances of the traverse lines using a systematic mathematical method so the adjusted measurements close. The "Bowditch rule" or "compass rule" in geodetic science and surveying assumes that linear error is proportional to the length of sides in relation to the perimeter of the traverse. Allowable misclosure is decided upon a case by case basis.

- Closed traverse is useful in marking the boundaries of wood or lakes. Construction and civil engineers utilize this practice for preliminary surveys of proposed projects in a particular designated area. The terminal (ending) point closes at the starting point.

Compound

A compound traverse is where an open traverse is linked at its ends to an existing traverse to form a closed traverse. The closing line may be defined by coordinates at the end points which have been determined by previous survey. The difficulty is, where there is linear misclosure, it is not known whether the error is in the new survey or the previous survey.

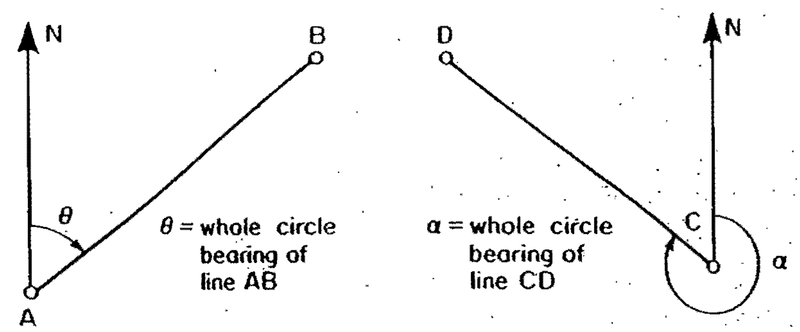
Usages

- **Control point** — The primary/base control used for preliminary measurements; it may consist of any known point capable of establishing accurate control of distance and direction (i.e. coordinates, elevation, bearings, etc.).

1. ***Starting*** – The initial starting control point of the traverse.
2. ***Observation*** – All known control points that are setted or observed within the traverse.
3. ***Terminal*** – The initial ending control point of the traverse; its coordinates are *unknown*.

General Traverse Specifications

TYPE	TYPICAL ACCURACY	PURPOSE	ANGULAR MEASUREMENT	DISTANCE MEASUREMENT
Geodetic	1 in 50 000 or better	(1) Major Control for mapping large areas (2) Provision of very accurate reference points for engineering surveys	0.1" theodolite	EDM
General	1 in 5000 to 1 in 50 000	(1) General engineering surveys, that is, setting out and site surveys (2) Secondary control for mapping large areas	1" or 20" theodolite	EDM, steel tapes, subtense methods
Low Accuracy	1 in 500 to 1 in 5000	(1) Small scale detail surveys (2) Rough large scale detail surveys (3) Preliminary or reconnaissance surveys	20" or 1' theodolite	Synthetic tapes, chains, stadia tacheometry



Whole-circle bearings

North Directions

The specified reference or north direction on which bearings are based may be true north, magnetic north, some entirely arbitrary direction assigned as north, or grid north.

True north

The accurate determination of this direction is undertaken only for special surveys. True north is not normally used in traversing for engineering surveys. However, an approximate value can be scaled from Ordnance Survey (OS) maps.

Magnetic north

This is determined by a freely suspended magnetic needle and can be measured with a prismatic compass.

A *prismatic compass* is a hand-held device which consists, basically, of a magnetic needle freely supported at its centre and usually immersed in oil to dampen oscillations. The needle will always indicate magnetic north although it can be unreliable in areas of strong local magnetic attraction and also when held near metal objects. For this reason, a prismatic compass must never be held against a metal ranging rod when taking readings.

The compass is graduated from 0° to 360° in half-degree intervals and is so designed that the magnetic bearing is obtained directly. The precision of the reading system is, at best, only ±15′ owing to difficulties in holding the compass steady and the rather crude sighting system.

Magnetic north is used only in reconnaissance surveys or to give a general indication of north when an arbitrary north is chosen for the survey.

Arbitrary north

Arbitrary north is most commonly used to define bearings in engineering traverses.

Any convenient direction is usually chosen to represent north even though it is not, in general, a true or magnetic north direction.

If a link traverse is being run between sets of known stations then the north direction may be determined from the values previously assigned to these known points.

Grid north

This north direction is based on the National Grid

Rectangular Coordinates

The coordinate system adopted for most survey purposes is a plane, rectangular system using two axes at right angles to one another as in Cartesian geometry. One is termed the north (N) axis and the other the east (E) axis. The scale along both axes is *always* the same.

With reference to figure 5.4, any particular point P has an easting (E) and a northing (N) coordinate, always quoted in the order easting, northing unless otherwise stated.

The position of each traverse station in a scheme in relation to all the others is specified in terms of these E and N coordinates.

Bearings are related to the north axis of the coordinate system.

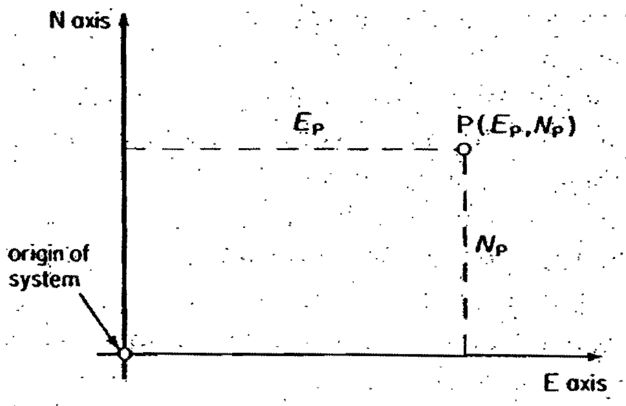


Figure 5.4 *Rectangular coordinate system*

For *all* types of survey and engineering works, the origin is taken at the extreme south and west of the area so that all coordinates are *positive*. If, at some stage in a survey, negative coordinates arise, the origin should be moved such that all coordinates will again be positive.

5.4 Traversing Fieldwork: Reconnaissance

5.4.1 General

This is one of the most important aspects of any surveying operation and must always be undertaken *before* any angles or lengths are measured.

(1) The main aim of the reconnaissance is to locate suitable positions for stations and hence a poorly executed reconnaissance can result in difficulties at later stages in the survey, leading to wasted time and inaccurate work.

(2) An overall picture of the area is obtained by walking all over the site (more than once is recommended), keeping in mind the requirements of the survey and balancing this against the accuracy and hence method of survey to be used. If an existing map or plan of the area is available, this is a useful aid at this stage.

(3) Where possible, *work from the whole to the part* as described in section 1.4, but an attempt should be made to keep the number of stations to a minimum.

(4) The lengths of traverse legs should be kept as long as possible to minimise the effect of any centring errors (see sections 3.3.4 and 5.5.1).

(5) If the traverse is being run for a detail survey then the method which is to be used for this subsequent operation must be considered.

For most sites a polygon traverse is usually sited around the perimeter of the area at points of maximum visibility. It should be possible to observe cross checks or lines across the area to enable other points inside the area to be fixed and also to assist in the location of angular errors.

Traverses for roadworks and pipelines generally require a link traverse since these sites tend to be long and narrow. The shape of the road or pipeline dictates the shape of the traverse.

(6) If the linear measurements are to be carried out using a tape or chain the ground conditions between stations should be suitable for this purpose. Try to avoid steep slopes or badly broken ground along the traverse lines. It is also better if there are as few changes of slope as possible. Roads and paths that have been surfaced are usually good for ground measurements.

(7) Stations should be located such that they are clearly intervisible, preferably at ground level, that is, with a theodolite set up at one point, it should be possible to see the ground marks at adjacent stations and as many others as possible. This eases the angular measurement process and enhances its accuracy.

(8) Stations should be placed in firm, level ground so that the theodolite and tripod are supported adequately when observing angles at the stations.

Very often stations are used for a site survey and at a later stage for setting out. Since some time may elapse between the site survey and the start of the construction the choice of firm ground in order to prevent the stations moving in any way becomes even more important. It is sometimes necessary to install semi-permanent stations (see section 5.4.2).

(9) Owing to the effects of lateral refraction and shimmer, traverse lines of sight should be well above ground level (greater than 1 m) for most of their length to avoid any possible angular errors due to rays passing close to ground level (*grazing rays*). These effects are serious in hot weather.

(10) When the stations have been sited, a sketch of the traverse should be prepared *approximately to scale*. The stations are given reference letters or numbers. This greatly assists in the planning and checking of fieldwork.

5.4.2 Station Marking

When a reconnaissance is completed, the stations have to be marked for the duration, or longer, of the survey.

Station markers must be permanent, not easily disturbed and they should be clearly visible. The construction and type of station depends on the requirements of the survey.

For general purpose traverses, wooden pegs are used which are hammered into the ground until the top of the peg is almost flush with ground level. If it is not possible to drive the whole length of the peg into hard ground the excess above the ground should be sawn off. This is necessary since a long length of peg left above the ground is liable to be knocked. A nail should be tapped into the top of the peg to define the exact position of the station.

Figure 5.5 shows such a station. Several months use is possible with this type of marker.

Stations in roadways can be marked with 75 mm pipe nails driven flush with the surface. The nail surround should be painted for easy identification. These marks are fairly permanent, but it is usually prudent to enquire if the road is to be resurfaced in the near future.

A more permanent station would normally require marks set in concrete; common station designs are shown in figure 14.2. These have to be placed with the permission of land owners as subsurface concrete blocks placed in a field could do considerable damage to farm machinery

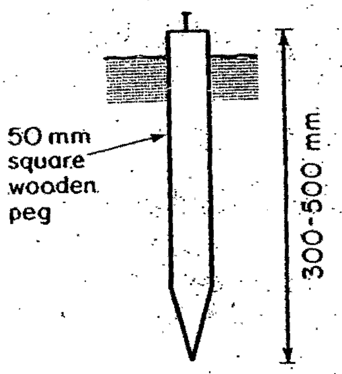


Figure 5.5 *Station peg*

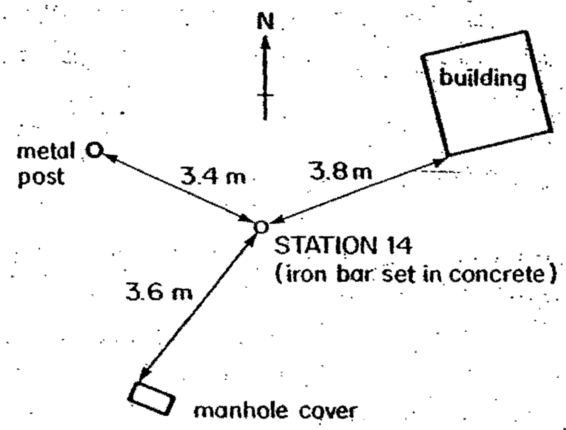


Figure 5.6 *Witnessing sketch*

A *reference* or *witnessing sketch* of the features surrounding each station should be prepared, especially if the stations are to be left for any time before being used, or if they will be required again at a much later stage. Measurements are taken from the station to nearby permanent features to enable it to be relocated. A typical sketch is shown in figure 5.6.

5.5 Traversing Fieldwork: Angular Measurement

Once the traverse stations have been placed in the ground the next stage in the field procedure is to use a theodolite to measure the included angles between the lines.

This requires two basic operations: setting the theodolite over each station mark, and observing the directions to the required stations.

In most cases it will be necessary to provide a signal at the observed stations since the station marks may not be directly visible. The theodolite and signals have to be erected perpendicularly above the station marks, otherwise centring errors will result.

5.5.1 Centring Errors

The measurement of traverse angles requires that the theodolite and signals be located in succession at each station. If this operation is not carried out accurately, centring errors are introduced, the effect of which depends on the length of the traverse leg, as discussed in section 3.3.4.

If a target displacement of 10 mm occurs on a 300 m traverse leg, the resulting angular error is 7″. The same displacement on a 30 m leg will produce an angular error of 70″. If this occurred during a traverse, the error would be carried through the rest of the traverse, and all subsequent bearings would be incorrect.

Hence, the effect of relatively small centring errors can be serious on short traverse legs.

If the theodolite is also displaced a further source of error arises.

The conclusion is that with both theodolites and signals care in centring is vital, especially when traverse legs are short.

5.5.2 Station Signals

Signals should be perfectly straight objects set up vertically and centred exactly over the station mark.

If a signal is not vertical then a centring error will be introduced, even though the base of the signal may be centred accurately over the mark. This is demonstrated by figure 5.7.

From figure 5.7, the lower the point of observation on the target, the smaller will be the centring error. For this reason, the lowest visible point on any signal should always be observed when measuring angles. However, this does not apply to special traverse targets (see section 5.7).

A subject not often considered is the width or diameter of a signal. It is a waste of time trying to observe a direction to a ranging rod when the line of sight is only 30 m, since accurate bisection is difficult to achieve. The width of a target should be proportional to the length of sight. Suggestions for some simple types of target are as follows.

(1) The station mark should be observed directly if possible. This can often be the case over short lines if the mark is a nail in the top of a wooden peg.

(2) If the mark cannot be seen directly, a pencil held on the mark can be used for convenience.

(3) A marking arrow can be held with its point on the mark or inserted in the top of the peg on the line of sight directly behind the nail.

(4) A tripod can be set up such that a plumb bob can be suspended from it directly over the mark. The plumb line can then be observed. Care must be exercised to ensure that the plumb bob does not rest on the mark as the string will then no longer be vertical, as shown in figure 5.8.

(5) For longer lines, ranging poles can be used. These must be carefully centred over the mark and must be held vertically by hand or in a ranging rod stand. The lowest part of the rod must be observed.

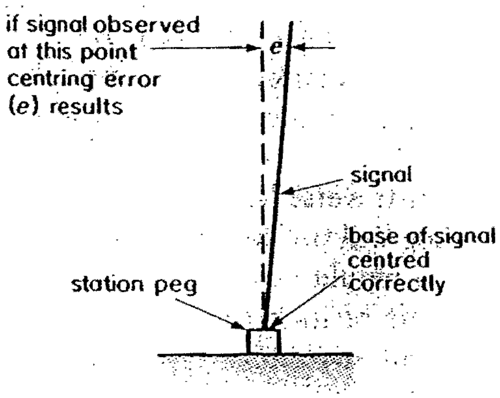


Figure 5.7 *Signal not vertical*

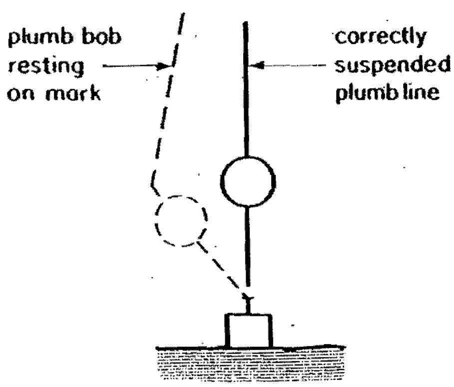


Figure 5.8 *Plumb line not vertical*

5.5.3 Abstraction of Angles

The general case at any station in a traverse is that the angle to be measured will be between some signal at a *back station* and some signal at a *forward station*, as shown in figure 5.9.

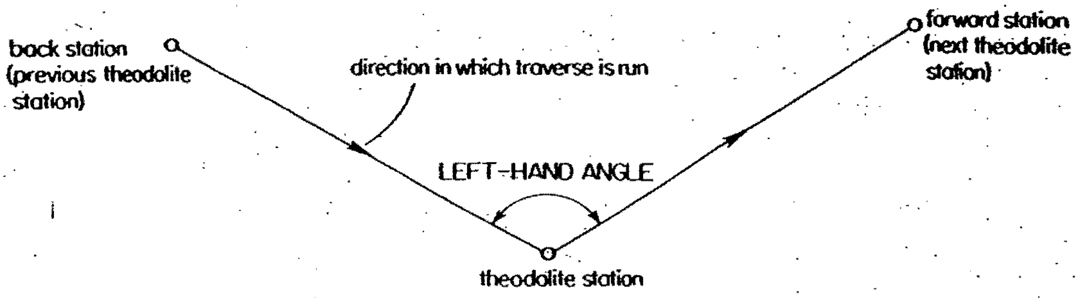


Figure 5.9 *Left-hand angle*

When readings have been taken to both stations, the angle abstracted may be either

(1) *left-hand angle* = mean forward circle reading − mean back circle reading

or

(2) *right-hand angle* = mean back circle reading − mean forward circle reading.

For computations, *either* can be chosen, but, for a particular traverse, it is important that the *same* angle should be abstracted at every station.

It is conventional, however, that the left-hand angle is chosen, the reason being that in computations the left-hand angle is added to the bearing to the back station to give the bearing to the forward station. If the right-hand angle is abstracted, this would have to be subtracted, an operation that is more liable to error (see section 5.8.3).

For polygon traverses, the *internal* angles of the polygon will be the left-hand angles if fieldwork proceeds in an *anticlockwise* direction around the traverse.

5.5.4 Field Procedure and Booking

The method given in chapter 3 for the reading and booking of angles should be adhered to whenever possible.

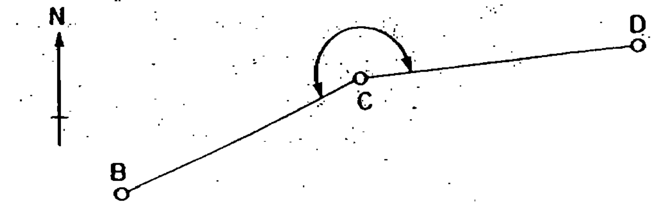
In the case where no standard booking forms are available, the angles can be entered in a field book, as in figure 5.10, in which *two complete rounds* of angles have been observed and the *zero changed* between rounds. The reasons for this are discussed in section 3.3.2.

AT STATION C

STATION		FL	FR	MEAN	ANGLE
1	B	00° 07' 20"	180° 07' 10"	00° 07' 20"	
	D	192° 23' 40"	12° 23' 20"	192° 23' 30"	192° 16' 10"
2	B	87° 32' 40"	267° 32' 20"	87° 32' 30"	
	D	279° 49' 20"	99° 49' 00"	279° 49' 10"	192° 16' 40"

MEAN ANGLE = 192° 16' 30"

DIAGRAM



OBSERVER WFP

BOOKER JU DATE 5th September 1985

Figure 5.10 *Booking traverse angles*

5.5.5 Errors in Angular Measurements

The various sources of error that may arise when measuring traverse angles are summarised as follows.

(1) Inaccurate centring of the theodolite or signal.
(2) Nonverticality of the signal.
(3) Inaccurate bisection of the signal.
(4) Parallax not eliminated.
(5) Lateral refraction, wind and atmospheric effects.
(6) Theodolite not level and not in adjustment (see sections 3.3.3 and 3.5).
(7) Incorrect use of the theodolite.
(8) Mistakes in reading and booking.

5.6 Traversing Fieldwork: Distance Measurement

For the purposes of traversing, distance measurement of the traverse legs is normally undertaken using steel taping or EDM, both of which are discussed in chapter 4.

5.7 The Three-tripod System

Very often, short traverse legs are unavoidable, for example, in surveys in mines and tunnels and on congested sites. Some manufacturers provide special equipment for

use on short traverse lines and it is known as the three-tripod system. This type of equipment is now used extensively in ordinary traverse work.

As described in section 3.2.1, the main feature of the system is that the body of the theodolite can be lifted from the tribrach and replaced by a special target. Thus, with the use of three or more tripods, the theodolites and targets can occupy the same positions and centring errors are greatly reduced. Distance measuring equipment can also be placed in the tribrachs and linear measurements are therefore made between exactly the same points as angular measurements.

The system operates as follows, with reference to figure 5.11.

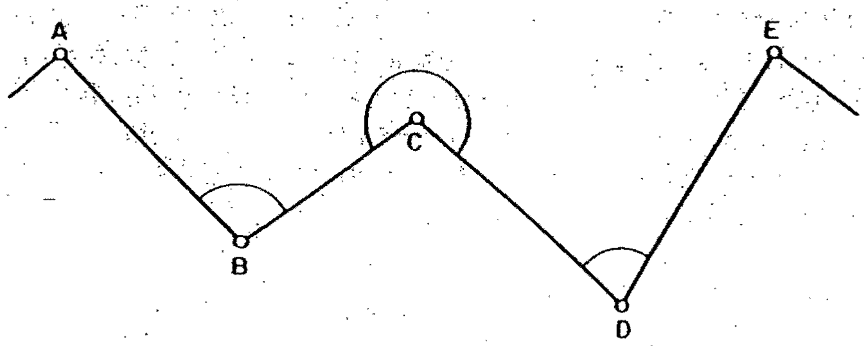


Figure 5.11 *Three-tripod traversing*

When angle ABC is measured

(1) At A a tripod is set up and a tribrach attached to the tripod head. A special target is placed into the tribrach and clamped in position. The target or tribrach will have a tube or pond bubble attached so that the target can be set vertical by levelling using the tribrach footscrews. In order to be able to centre the target, the tribrach usually has an optical plummet.

(2) At B the theodolite is set up in the normal manner.

(3) At C a tripod and target is set up as at A.

When angle BCD is measured

(1) At A the tripod and target are moved to D, where the target is again centred and set vertical.

(2) At B the theodolite is unclamped, removed from its tribrach and interchanged with the target at C. Hence, at B and C, the tripods and tribrachs remain undisturbed and there is no need for recentring.

When angle CDE is measured

(1) At B the tripod and target are moved to E.

(2) The theodolite and target at C and D are interchanged, the tribrachs (and centring) remaining undisturbed.

The process is repeated for the whole traverse. If *four* tripods (or more) are used this speeds up the fieldwork considerably as tripods can be moved and positioned while angles are being measured.

Three tripod systems are more expensive than basic systems and more equipment has to be moved around the site by the engineer and his assistants. However, the advantages of the dramatic reduction of centring errors and a saving in time far outweigh these disadvantages.

5.8 Traversing Calculations

5.8.1 Abstract of Fieldwork

When all the traverse fieldwork has been completed, a single sheet or record containing the mean angles observed and mean horizontal (corrected) lengths measured should be prepared. It is preferable to show all the data on a sketch of the traverse as this helps in the following calculations and can minimise the chance of a mistake.

Such an abstraction of field data is shown in figure 5.12, the angles and lengths being entered on to a traverse diagram. The example shown in figure 5.12 will be referred to through section 5.8.

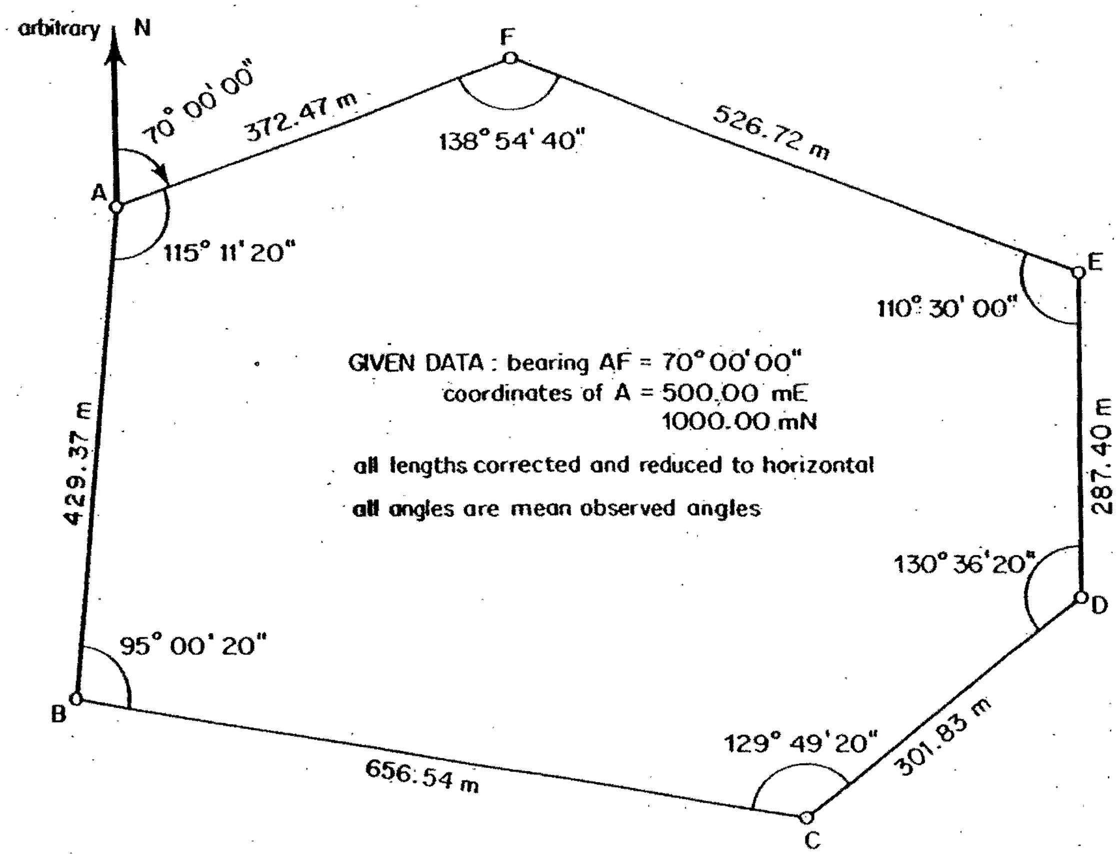


Figure 5.12 *Traverse diagram*

5.8.2 Angular Misclosure

Determination of misclosure

For a closed traverse, before any coordinate calculations can commence, the whole circle bearings of all the lines have to be calculated, the initial stage in the process

being to check that the observed angles sum to the required value.

The observed angles of a *polygon traverse* can be either the *internal* or *external* angles; whichever is abstracted depends on the direction in which the traverse is run (see section 5.8.3).

The angular misclosures are found by comparing the sum of the observed angles with one of the following theoretical values.

(1) Sum of *internal* angles = $(2n - 4) \times 90°$

or (2) Sum of *external* angles = $(2n + 4) \times 90°$

where n is the number of angles or sides of the polygon.

For the traverse, the observed angles are summed and a check is made according to one of the above formulae.

When the bearings in a *link traverse* are calculated, an *initial back bearing* (see section 5.8.3) can usually be determined from known points at the start of the traverse and, to check the observed angles, a *final forward bearing* (see section 5.8.3) is computed from known points at the end of the traverse. The method for obtaining a bearing from coordinates is given in section 5.10. The angular misclosure in a link traverse is found by using the following theoretical relationship

$$\text{sum of } \textit{left-hand angles} = (\text{final forward bearing} - \text{initial back bearing}) + (n - 1) \times 180°$$

where n is the number of left-hand angles measured.

For both types of traverse, care must be taken to ensure that the correct angles have been abstracted and summed, that is, the internal *or* external angles in a polygon traverse and the left-hand angles in a link traverse. When the angles have been summed and checked, a very large misclosure probably means that an incorrect angle has been included, or one of the angles has been excluded.

Allowable misclosure

Owing to the effects of occasional miscentring, slight misreading and small bisection errors, a small misclosure will result when the summation check is made.

The allowable misclosure (E) is $E'' = \pm KS(N)^{\frac{1}{2}}$ where N is the number of traverse stations, S the smallest reading interval on the theodolite in seconds, for example, 60″, 20″, 1″, and K the multiplication factor of 1 to 3, depending on weather conditions, number of rounds taken, and so on. The allowable misclosure for the traverse shown in figure 5.12 varies from approximately 50″ to 150″ (assuming a 20″ theodolite was used).

Adjustment

When the actual misclosure is known and is compared to its allowable value, two cases may arise.

(1) *If the misclosure is acceptable* (less than the allowable) it is divided *equally* between the left-hand angles. An equal distribution is the only acceptable method since each angle is measured in the same way and there is an equal chance of the misclosure having occurred in any of the angles.

No attempt should be made to distribute the misclosure in proportion to the size of an angle.

(2) *If the misclosure is not acceptable* (greater than the allowable) the angles should be remeasured if no gross error can be located in the angle bookings or summation.

It may be possible to isolate a gross error in a small section of the traverse if check lines have been observed across it.

Example of angular misclosure and adjustment

The determination of the misclosure and adjustment of the angles of the polygon traverse given in figure 5.12 is shown in table 5.2.

Example 5.12.2 at the end of this chapter shows how the angles in a link traverse are adjusted.

5.8.3 Calculation of Whole-circle Bearings

Types and determination of bearings

Consider figure 5.13, which shows two legs of a traverse. The decision has been made to calculate the traverse in the direction . . . X to Y to Z . . . This defines the bearings as follows.

Bearings XY and YZ are *forward bearings* since they are in the same direction in which calculations are proceeding.

Bearings YX and ZY are *back bearings* since they are opposite to the direction in which the traverse calculation is proceeding.

Directions ZY and YZ differ by $\pm 180°$, as do those of YX and XY. Therefore, the forward bearing of a line differs from the back bearing by $\pm 180°$. Since whole-circle bearings must lie in the range $0°$ to $360°$, some multiple of $360°$ is either subtracted from or added to a forward or back bearing outside this range to bring the resulting bearing into the range $0°$ to $360°$. For example

$$\text{a bearing of } 520° = \text{a bearing of } (520 - 360)° = 160°$$

$$\text{a bearing of } -200° = \text{a bearing of } (-200 + 360)° = 160°$$

Bearings are calculated relative to the selected north line, usually starting from a given or assumed bearing for one line.

If bearing YX was known, this could be drawn as shown in figure 5.13 relative to the selected north line.

For the direction of computation shown in figure 5.13, the *left-hand angle* γ_Y has been abstracted and the known bearing YX is a *back bearing*.

If γ_Y is added to the back bearing YX it can be seen from figure 5.13 that the resulting angle will be the forward bearing YZ. Thus

$$\text{forward bearing YZ} = \text{back bearing YX} + \gamma_Y$$

Therefore, in general, for any particular traverse station

$$\textit{forward bearing} = \textit{back bearing} + \textit{left-hand angle}$$

For polygon traverses when working in an *anticlockwise* direction around the

TABLE 5.2

STATION	LEFT HAND ANGLE	ADJUSTMENT	ADJUSTED LEFT HAND ANGLE
A	115° 11' 20"	- 20"	115° 11' 00"
B	95° 00' 20"	- 20"	95° 00' 00"
C	129° 49' 20"	- 20"	129° 49' 00"
D	130° 36' 20"	- 20"	130° 36' 00"
E	110° 30' 00"	- 20"	110° 29' 40"
F	138° 54' 40"	- 20"	138° 54' 20"
Sums	720° 02' 00"	- 02' 00"	720° 00' 00"

Required Sum = ((2x6) - 4) x 90
= 720° 00' 00"
Misclosure = + 02' 00"

Adjustment per angle
= - (02' 00")/6
= - 20"

traverse, the left-hand angles will be the *internal* angles of the traverse and when working in a *clockwise* direction, the left-hand angles will be the *external* angles.

Either clockwise or anticlockwise can be run since abstracting the left-hand angles will always give the correct angles for the bearings computations.

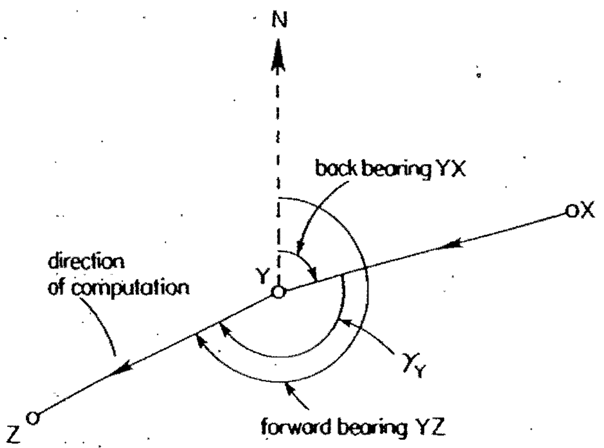


Figure 5.13 ***Whole-circle bearing calculation***

Example of bearing calculation

Some of the bearings of the lines of the traverse shown in figure 5.12 will now be computed using adjusted left-hand angles. Figures 5.14 and 5.15 show sections of this traverse.

At station A in figure 5.14

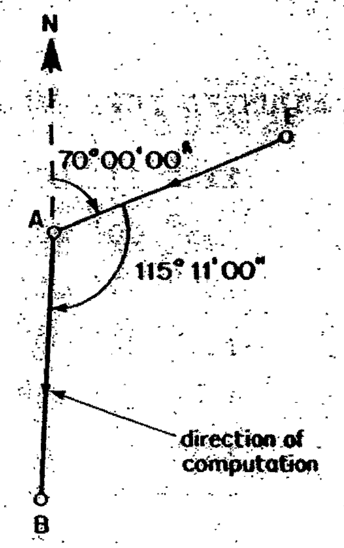


Figure 5.14

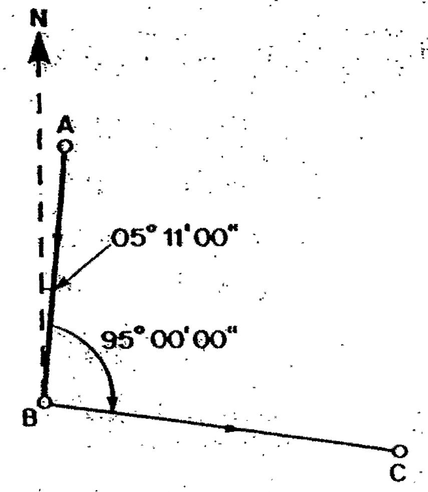


Figure 5.15

forward bearing AB = back bearing AF + left-hand angle at A

$= 70° \ 00' \ 00''$ (given) $+ \ 115° \ 11' \ 00''$

$= 185° \ 11' \ 00''$

At station B in figure 5.15

forward bearing BC = back bearing BA + left-hand angle at B

But back bearing BA = forward bearing AB ± 180°

$= 185° \ 11' \ 00'' \pm 180°$

$= 365° \ 11' \ 00''$ *or* $05° \ 11' \ 00''$

$= 05° \ 11' \ 00''$ (to keep bearing in range 0 to 360°)

Hence *forward bearing* BC = $05° \ 11' \ 00'' + 95° \ 00' \ 00''$

$= 100° \ 11' \ 00''$

The bearings of all the lines can be computed in a similar manner; the complete calculation is given in table 5.3.

Every bearing calculation finishes by recalculating the initial (given) bearing. This final computed bearing must be in agreement with the initial bearing and, if any difference occurs, an *arithmetic mistake* has been made, and the bearing calculation must be checked before proceeding to the next stage in the calculation.

5.8.4 Computation of Coordinate Differences

The next stage in the traverse computation is the determination of the coordinate differences of the traverse lines.

The information available at this point will be the bearings and horizontal lengths of all the lines.

TABLE 5.3

LINE / STATION / LINE	BACK BEARING / ADJUSTED LEFT HAND ANGLE / FORWARD BEARING			WHOLE CIRCLE BEARING θ			HORIZONTAL DISTANCE D	COORDINATE DIFFERENCES: CALCULATED ΔE	ΔN	ADJUSTMENTS δE	δN	ADJUSTED ΔE	ΔN	COORDINATES E	N	STATION
														500.00	1000.00	A
AF	70	00	00													
A	115	11	00													
AB	185	11	00	185	11	00	429.37	-38.79	-427.61	-0.04	+0.02	-38.83	-427.59	461.17	572.41	B
BA	05	11	00													
B	95	00	00													
BC	100	11	00	100	11	00	656.54	+646.20	-116.08	-0.05	+0.03	+646.15	-116.05	1107.32	456.36	C
CB	280	11	00													
C	129	49	00													
CD	50	00	00	50	00	00	301.83	+231.22	+194.01	-0.03	+0.01	+231.19	+194.02	1338.51	650.38	D
DC	230	00	00													
D	130	36	00													
DE	00	36	00	00	36	00	287.40	+3.01	+287.38	-0.02	+0.01	+2.99	+287.39	1341.50	937.77	E
ED	180	36	00													
E	110	29	40													
EF	291	05	40	291	05	40	526.72	-491.42	+189.57	-0.04	+0.03	-491.46	+189.60	850.04	1127.37	F
FE	111	05	40													
F	138	54	20													
FA	250	00	00	250	00	00	372.47	-350.01	-127.39	-0.03	+0.02	-350.04	-127.37	500.00	1000.00	A
							Σ 2574	Σ+0.21	Σ-0.12	Σ-0.21	Σ+0.12	Σ 0.00	Σ 0.00			

ACTUAL SUM OF LEFT HAND ANGLES = 720° 02' 00"

REQUIRED SUM OF LEFT HAND ANGLES = (2 X 6 - 4) X 90° = 720°

MISCLOSURE = + 02' 00"

ADJUSTMENT TO EACH OBSERVED ANGLE = - 20"

$e_E = +0.21$ ADJUSTMENT TO $\Delta E/\Delta N$ BY BOWDITCH

$e_N = -0.12$

$e = ((+0.21)^2 + (-0.12)^2)^{\frac{1}{2}} = 0.24$

FRACTIONAL LINEAR MISCLOSURE = 1 in 10 700

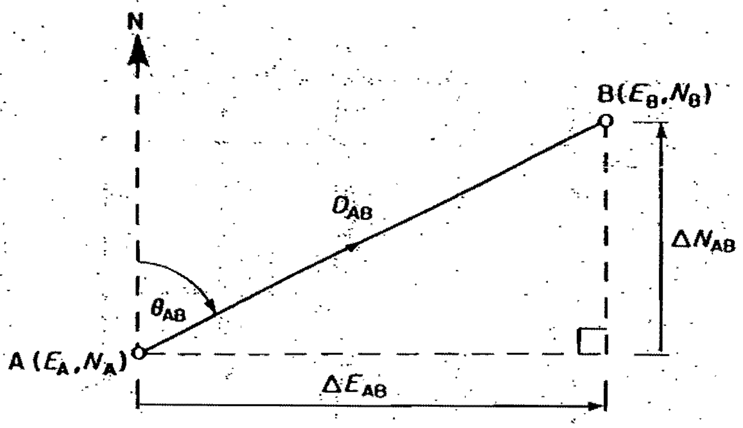


Figure 5.16 *Coordinate differences*

Coordinate differences

The coordinate differences are computed as shown in figure 5.16. Coordinates of station A are E_A, N_A which are known. Coordinates of station B are E_B, N_B which are to be calculated.

θ_{AB} = whole circle bearing of line AB

D_{AB} = horizontal length of line AB

ΔE_{AB} = *eastings difference* in moving from station A to station B

ΔN_{AB} = *northings difference* in moving from station A to station B

With reference to figure 5.16

$$E_B = E_A + \Delta E_{AB} = E_A + D_{AB} \sin \theta_{AB} \tag{5.1}$$

$$N_B = N_A + \Delta N_{AB} = N_A + D_{AB} \cos \theta_{AB} \tag{5.2}$$

For the traverse, each line is considered separately and the coordinate differences ΔE and ΔN computed.

If a pocket calculator is used, values of ΔE and ΔN can be obtained directly from equations (5.1) and (5.2) for any value of θ. Alternatively, the *polar/rectangular key* found on most calculators can be used. Since the method by which this is achieved depends on the make of calculator, the handbook supplied with the calculator should be consulted.

Examples of coordinate difference calculation

The traverse data of figure 5.12 are again used in the following examples. The bearings are the whole-circle bearings given in table 5.3.

Consider line AB in both figure 5.12 and figure 5.17.

From equation (5.1)

$$\begin{aligned}\Delta E_{AB} &= D_{AB} \sin \theta_{AB}\\ &= 429.37 \sin 185° \ 11' \ 00'' = 429.37\ (-0.09034)\\ &= -38.79 \text{ m}\end{aligned}$$

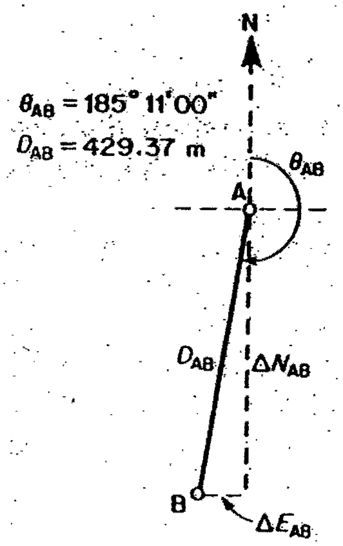


Figure 5.17

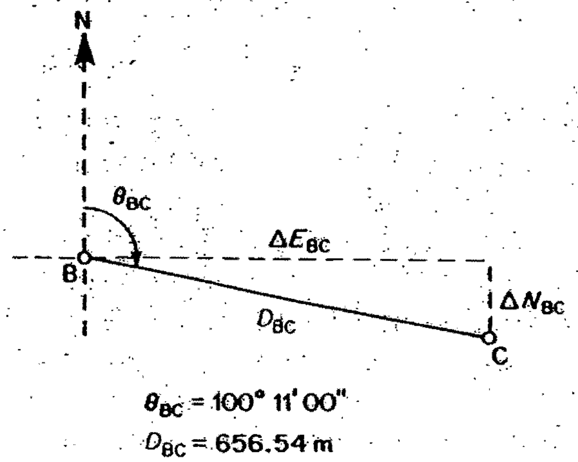


Figure 5.18

Similarly, from equation (5.2)

$$\begin{aligned}\Delta N_{AB} &= D_{AB} \cos \theta_{AB} \\ &= 429.37 \cos 185° 11' 00'' = 429.37\ (-0.99591) \\ &= -427.61 \text{ m}\end{aligned}$$

For line BC shown in figures 5.12 and 5.18, the coordinate differences are given by

$$\begin{aligned}\Delta E_{BC} &= D_{BC} \sin \theta_{BC} \\ &= 656.54 \sin 100° 11' 00'' = 656.54\ (+0.98425) \\ &= +646.20 \text{ m}\end{aligned}$$

$$\begin{aligned}\Delta N_{BC} &= D_{BC} \cos \theta_{BC} \\ &= 656.54 \cos 100° 11' 00'' = 656.54\ (-0.17680) \\ &= -116.08 \text{ m}\end{aligned}$$

As with the bearing calculations, the coordinate difference results are always presented in tabular form since errors are easier to detect.

For the traverse ABCDEFA (figure 5.12) all the calculations for coordinate differences are given in table 5.3.

5.8.5 Misclosure

When the ΔE and ΔN values have been computed for the whole traverse as in table 5.3, checks can be applied to the computation.

For *polygon* traverses these are

$$\Sigma \Delta E = 0 \text{ and } \Sigma \Delta N = 0$$

since the traverse starts and finishes at the same point.

For link traverses (figure 5.1) these are

$$\Sigma\Delta E = E_Y - E_X \quad \text{and} \quad \Sigma\Delta N = N_Y - N_X$$

where station X is the starting point and station Y the final point of the traverse. Since stations X and Y are of known position, the values of $E_Y - E_X$ and $N_Y - N_X$ can be calculated.

In both cases, owing to field errors in measuring the angles and lengths, there will normally be a misclosure on returning to the starting point on a polygon traverse or on arrival at the final known station in a link traverse.

This *linear* misclosure is computed and any adjustment is allocated appropriately.

Therefore, before the station coordinates are calculated, the ΔE and ΔN values found for the traverse are summed and the misclosures, e_E and e_N, are found by comparing the summations with those expected.

These misclosures form a measure of the linear misclosure of the traverse and can be used to determine the accuracy of the survey. Consider figure 5.19 which shows the starting point A of a polygon traverse.

Owing to field errors, the traverse ends at A′ instead of A. The *linear misclosure*, e, is given by

$$e = (e_E{}^2 + e_N{}^2)^{\frac{1}{2}}$$

To obtain a measure of the accuracy of the traverse, this misclosure is compared with the total length of the traverse legs, ΣD, to give the *fractional linear misclosure*, where

fractional linear misclosure = 1 in ($\Sigma D/e$) (for example, 1 in 10 000)

This fractional misclosure is *always* computed for a traverse and is compared with the value required for the type of survey being undertaken. For appropriate values of the fractional linear misclosure see table 5.1.

If, on comparison, the fractional linear misclosure is better than the required value, the traverse fieldwork is satisfactory and the misclosures, e_E and e_N are distributed throughout the traverse.

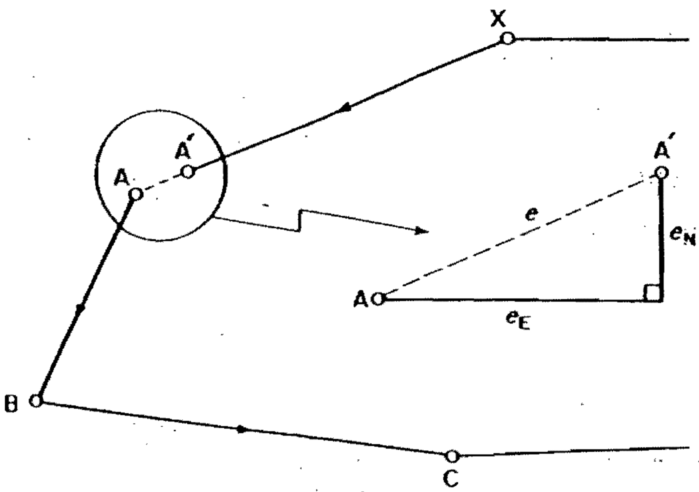


Figure 5.19 *Traverse misclosure*

If, on comparison, the fractional linear misclosure is worse than that required, there is most likely an error in the measured lengths of one or more of the legs. The calculations should, however, be thoroughly checked before remeasuring any lengths.

An example determination of the fractional linear misclosure can be obtained from table 5.3, remembering that the traverse is a polygon. From the table

$$(1)\ \Sigma\Delta E = -38.79 + 646.20 + 231.22 + 3.01 - 491.42 - 350.01$$
$$= +\ 0.21\ \text{m}$$

Hence

$$e_E = +\ 0.21\ \text{m since } \Sigma\Delta E \text{ should be zero}$$

$$(2)\ \Sigma\Delta N = -427.61 - 116.08 + 194.01 + 278.38 + 189.57 - 127.39$$
$$= -\ 0.12\ \text{m}$$

Hence

$$e_N = -\ 0.12\ \text{m since } \Sigma\Delta N \text{ should also be zero}$$

Therefore, *linear misclosure* $= e = [(+0.21)^2 + (-0.12)^2]^{\frac{1}{2}} = 0.24$ m. From figure 5.12 and table 5.3 $\Sigma D = 2574$ m.

Therefore, *fractional linear misclosure* = 1 in (2574/0.24) $\approx$ 1 in 10 700.

This procedure is shown at the bottom of table 5.3.

5.8.6 Distribution of the Misclosure

Many methods of adjusting the linear misclosure of a traverse are possible but, for everyday engineering traverses of accuracy up to 1 in 20 000, one of two methods is normally used.

Bowditch method

The values of the adjustment found by this method are directly proportional to the length of the individual traverse lines.

Adjustment to ΔE (or ΔN) for one particular traverse leg

$$= \delta E \text{ (or } \delta N) = -e_E \text{ (or } -e_N) \times \frac{\text{length of traverse leg concerned}}{\text{total length of the traverse}}$$

Transit method

In this method, adjustments are proportional to the values of ΔE and ΔN for the various lines.

Adjustment to ΔE (or ΔN) for one particular traverse leg

$$= \delta E \text{ (or } \delta N) = -e_E \text{ (or } -e_N) \times \frac{\Delta E \text{ (or } \Delta N) \text{ of the traverse leg concerned}}{\text{absolute } \Sigma\Delta E \text{ (or } \Sigma\Delta N) \text{ for the traverse}}$$

For both methods, the negative signs are necessary since if e_E (or e_N) is positive, the *adjustments* will be negative, and if e_E (or e_N) is negative the *adjustments* will be positive.

For the *Bowditch* method, the adjustment of the values of ΔE and ΔN given in table 5.3 is as follows.

The misclosures have already been determined as $e_E = +0.21$ m and $e_N = -0.12$ m, and the total length of the traverse is 2574 m.

For line AB

$$\delta E_{AB} = -0.21 \times (429/2574) = -0.04 \text{ m}$$

$$\delta N_{AB} = +0.12 \times (429/2574) = +0.02 \text{ m}$$

For line BC

$$\delta E_{BC} = -0.21 \times (657/2574) = -0.05 \text{ m}$$

$$\delta N_{BC} = +0.12 \times (657/2574) = +0.03 \text{ m}$$

This process is repeated for the whole traverse. These adjustments, applied to the ΔE and ΔN values, would normally be tabulated as shown in table 5.3.

Applying the *transit* method to the same example gives

$$\text{absolute } \Sigma\Delta E = 1761 \text{ m and absolute } \Sigma\Delta N = 1342 \text{ m}$$

Hence *for line AB*

$$\delta E_{AB} = -0.21 \times (39/1761) = -0.00 \text{ m}$$

$$\delta N_{AB} = +0.12 \times (428/1342) = +0.04 \text{ m}$$

and *for line BC*

$$\delta E_{BC} = -0.21 \times (646/1761) = -0.08 \text{ m}$$

$$\delta N_{BC} = +0.12 \times (116/1342) = +0.01 \text{ m}$$

Again, the computation is repeated for each line of the traverse.

Both the transit method and Bowditch's method will alter the original bearings by a very small amount. It is *not* necessary to recalculate these bearings unless the traverse is to be used for subsequent control work such as setting out.

Checks on both methods of adjustment should be undertaken as follows. If the adjustment has been carried out successfully

$$\Sigma\delta E \textit{ should} = -e_E$$

$$\Sigma\delta N \textit{ should} = -e_N$$

These checks must be carried out *before* calculating the adjusted ΔE and ΔN values.

5.8.7 Calculation of the Final Coordinates

For *polygon* traverses, in order to compute the coordinates of the stations, the coordinates of the starting point have to be known. These starting coordinates may either be assumed for an area to give positive coordinates for the whole survey or,

occasionally, may be given if a previously coordinated station is used to start the traverse.

For *link* traverses, the coordinates of the starting and finishing points will be known from a previous survey and the coordinates will be determined relative to these known values.

The coordinates of each point are obtained by adding or subtracting the adjusted ΔE and ΔN values as necessary, working around the traverse.

When all the coordinates have been calculated, there is a final check to be applied.

For a *polygon* traverse, the final and initial coordinates should be equal as these represent the same station.

For a *link* traverse, the final coordinates should equal those of the second known point.

If this check does not hold, there is an *arithmetical mistake* and the calculations should be investigated until it is found.

At this stage for the polygon traverse which has been referred to throughout this discussion (that shown in figure 5.12), the adjusted ΔE and ΔN values have now been determined and, since the coordinates of the starting point, station A, have been given as 500.00 mE and 1000.00 mN, the coordinates of the other traverse stations can be obtained from these initial coordinates and the adjusted ΔE and ΔN values found by the Bowditch method. For example

$$(1)\ E_B = E_A \pm \Delta E_{AB} = 500.00 - 38.83 = 461.17 \text{ m}$$

$$N_B = N_A \pm \Delta N_{AB} = 1000.00 - 427.59 = 572.41 \text{ m}$$

$$(2)\ E_C = E_B \pm \Delta E_{BC} = 461.17 + 646.15 = 1107.32 \text{ m}$$

$$N_C = N_B \pm \Delta N_{BC} = 572.41 - 116.05 = 456.36 \text{ m}$$

This process is repeated until station *A* is recoordinated as a check. The complete calculation is shown in table 5.3.

5.8.8 The Traverse Table

For each particular step in the traverse computation every calculation should be tabulated.

There are many variations of the layout that can be adopted but the format given in table 5.3 is recommended.

Table 5.3 shows the calculation for the polygon traverse ABCDEFA of figure 5.12. This table should be thoroughly studied, referring to the relevant preceding sections of this chapter to enable a complete understanding of how the table is compiled to be gained.

5.8.9 Precision of Computation

When using calculators and computers, care must be taken to use only the appropriate amount of the eight (sometimes more) significant figures presented by the display.

It must be realised that *any quantity calculated cannot be quoted to a higher precision than that of the data supplied or that of the field observations.*

Some examples are given to demonstrate this principle.

(1) In section 5.8.4, all the coordinate differences were calculated to the nearest 0.01 m.

Using a calculator, it would have been possible to compute, for example, ΔE_{AB} as -38.790524 m but without significance since the traverse legs were only measured to the nearest 0.01 m. Hence, the figure is rounded to -38.79 m since the coordinate differences can also, at best, be quoted only to the nearest 0.01 m.

(2) With reference to the example traverse of figure 5.12, the coordinates of station A were given as 500.00 mE and 1000.00 mN.

These are written in this manner to indicate that the position of station A is known to the nearest 0.01 m. Thus, all coordinates derived from this station can, at best, be quoted only to the nearest 0.01 m.

If the coordinates of station A were recorded as 500.0 mE and 1000.0 mN, this implies that the position of A is known only to the nearest 0.1 m. If this were the case, all subsequent coordinates can, at best, be quoted only to the nearest 0.1 m even though the coordinate differences can, perhaps, be quoted to 0.01 m.

When the coordinates are quoted as 500.000 mE and 1000.000 mN, it is now possible to quote coordinates of other stations to 0.001 m so long as the fieldwork techniques used warrant this.

However, since the ΔE and ΔN values in the example traverse are known only to the nearest 0.01 m, all the coordinates derived from those of station A can be determined only to 0.01 m, even if the coordinates of station A are quoted to 0.001 m.

This reaffirms the earlier statement that it is the least precise component in the calculations which determines the precision of the final result.

The above notes refer not only to traversing but also to any calculations in engineering.

5.9 Plotting Traverse Stations

When the final coordinates have been computed for a traverse, the stations have to be plotted if a site plan is being prepared.

A common mistake is to plot the stations using the bearings or left-hand angles and the lengths between stations. This is known as plotting by angle (or bearing) and distance. Such a method is NEVER used in traversing and the preferred and most accurate method of plotting traverse stations is to use the computed coordinates.

The methods given in the following sections for plotting traverse station coordinates apply also to triangulation and other stations (see chapters 6 and 7) when these have to be used in plan production.

5.9.1 Orientating the Survey and Plot

Before commencing any plot (that is, before constructing the grid), the extent of the survey should be taken into account such that the plotted survey will fall centrally on to the sheet.

In the case where the north direction is stipulated, the north–south and east–west extents of the area should be determined and the stations plotted for the best fit on the sheet.

If an arbitrary north is to be used, the best method of ensuring a good fit is to assign a bearing of 90° or 270° to the longest side. This line is then positioned parallel to the longest side of the sheet so that the survey will fit the paper properly.

Sometimes it may be necessary to set the arbitrary north to a particular direction in order to ensure that the survey will fit a particular sheet size. This will often be the case with long, narrow site surveys where, to save paper and for convenience, the plot of the survey is to go on to a single or a minimum number of sheets or the minimum length of a roll. The boundary of the survey should be roughly sketched and positioned until a suitable fit is obtained. Again, the longest or most convenient line should be assigned a suitable arbitrary bearing.

Where the north point is arbitrary, it has to be established before the coordinate calculation takes place. In order to estimate the extent of the survey it should be sketched, roughly to scale, using the left-hand angles and the lengths between stations.

5.9.2 Coordinate Grid

The first stage in plotting coordinates is to establish a coordinate grid.

Coordinated lines are drawn at specific intervals, for example, 10 m, 50 m, 100 m in both the east and north directions to form a pattern of squares. The stations are then plotted in relation to the grid.

When drawing the grid, T-squares and set squares should *not* be used since they are not accurate enough. Instead, the grid is constructed in the following manner.

(1) From each corner of the plotting sheet two diagonals are drawn, as shown in figure 5.20*a*.

(2) From the intersection of these diagonals an equal distance is scaled off along each diagonal using a *beam compass*. This scaled distance must be large, see figure 5.20*b*.

(3) The four marked points on the diagonals are joined using a *steel straight edge* to form a rectangle. This rectangle will be perfectly true and is used as the basis for the coordinate grid (see figure 5.20*c*).

On all site plans and maps it is conventional to have the north point (true, magnetic or arbitrary) on the drawing such that the north direction is from the bottom to the top of the sheet and roughly parallel to the sides of the sheet. This will be achieved if the grid framework is constructed as described.

(4) By scaling equal distances along the top and bottom lines of the rectangle and joining the points, the vertical (E) grid lines will be formed. The horizontal (N) grid lines are formed in a similar manner using the other sides of the rectangle (see figure 5.20*d* and *e*).

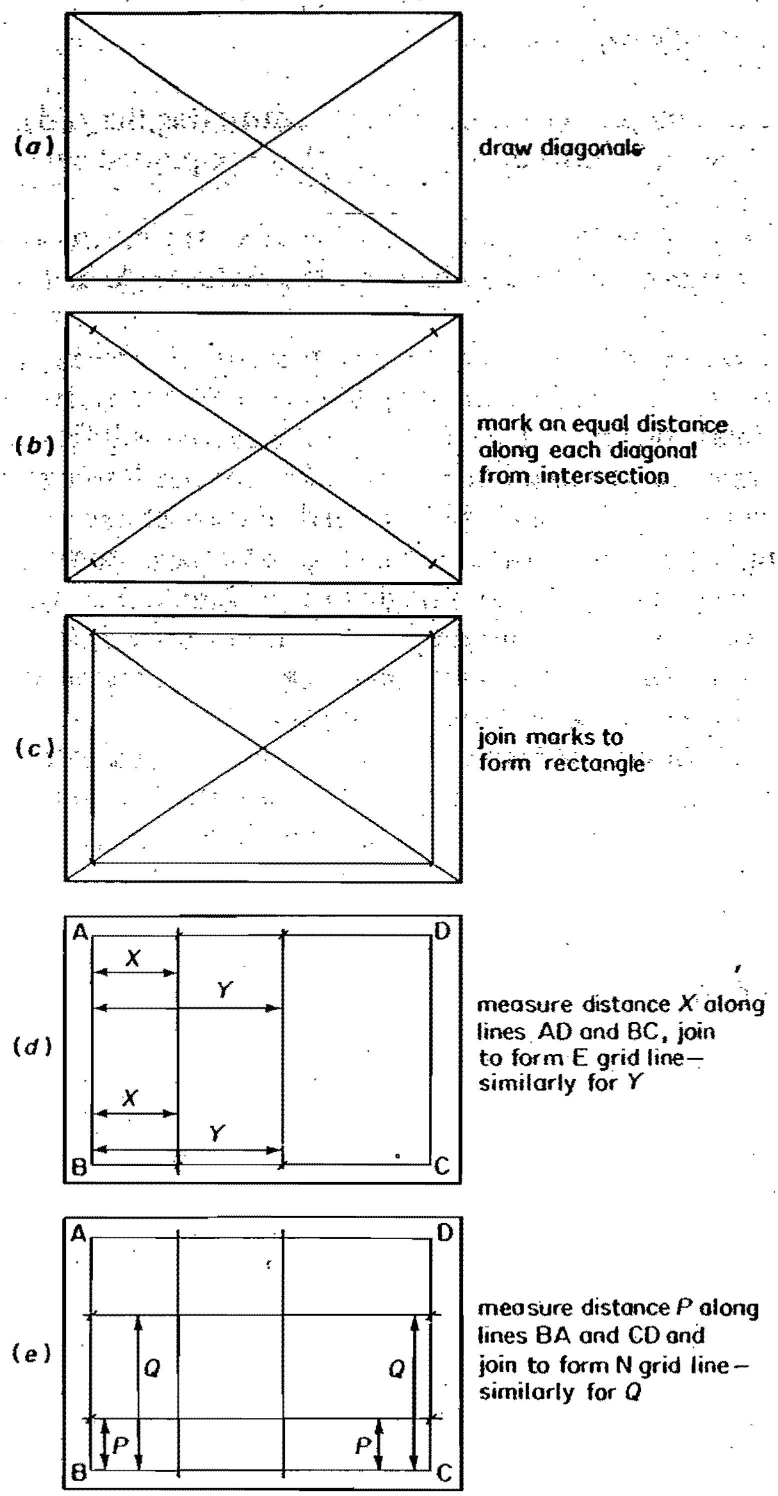


Figure 5.20 *Establishing coordinate grid*

All lines must be drawn with the aid of a steel straight edge and all measurements must be taken from lines AB and BC and not from one grid line to the next. This avoids accumulating errors.

(5) The grid lines should now be numbered accordingly. The size of a grid square should not be greater than 100 mm by 100 mm. It is not necessary to plot the origin of the survey if it lies outside the area concerned.

5.9.3 Station Plotting

When plotting by coordinates, the procedure is as follows.

Let the station to be plotted have coordinates 283.62 mE, 427.45 mN and let it be plotted on a 100 m grid previously prepared as described in section 5.9.2.

(1) The grid intersection 200 mE, 400 mN is located on the prepared grid.

(2) Along the 400 mN line, 83.62 m is scaled off from the 200 mE intersection towards the 300 mE intersection and point a is located (see figure 5.21). Similarly, point b is located along the 500 mN line. Points a and b are joined with a pencil line.

(3) Along the 200 mE line, 27.45 m is scaled from the 400 mN intersection towards the 500 mN intersection to locate point c. Point d is found by scaling 27.45 m along the 300 mE line. Points c and d are joined.

(4) The intersection of lines ab and cd gives the position of the station.

(5) To check the plotted position, dimensions X and Y are measured from the plot and compared with their expected values. In this case, X *should* equal $100.00 - 83.62 = 16.38$ m and Y *should* equal $100.00 - 27.45 = 72.55$ m.

(6) When all the stations have been plotted, the lengths between the plotted stations are measured and compared with their accepted values.

(7) The traverse lines are added by carefully joining the plotted stations. This is to aid in the location of detail (see section 8.5).

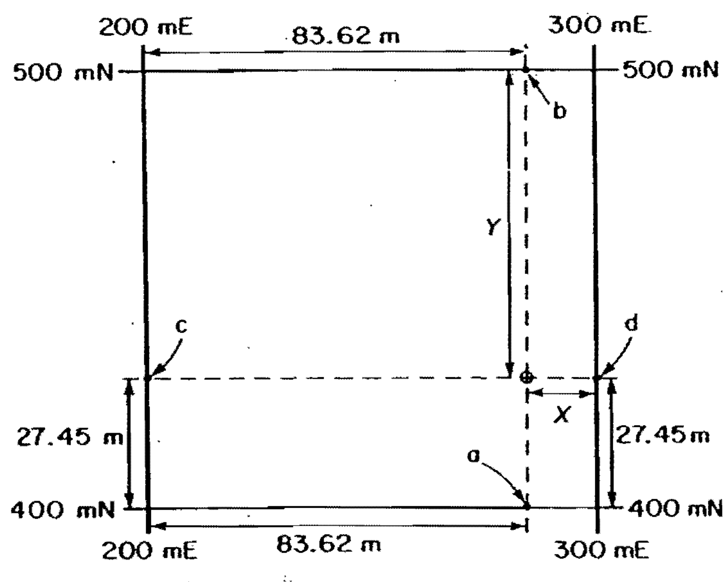


Figure 5.21 *Station plotting*

5.10 Whole-circle Bearing and Distance Calculation from Coordinates

In section 5.8.4 it was shown that, knowing the whole-circle bearing and length of a line, the coordinates of one end of the line could be computed if the coordinates of the other end were known.

For the reverse case, where the coordinates of two points are known, it is possible to compute the whole-circle bearing and the horizontal distance of the line between the two points.

This type of calculation is commonly used in engineering surveying when setting out works by polar coordinates. This is discussed in section 14.8.2.

In link traversing, it is often necessary to calculate the whole-circle bearings of the start and finish lines from the coordinates of the existing, known stations.

The WCBs and horizontal distances can be calculated by one of two methods; either by considering the *quadrant* in which the line falls or by using the *rectangular polar conversion key* found on most calculators. If this type of calculation is undertaken using a computer which does not have a rectangular/polar facility, any programs written must be based on the quadrants method.

5.10.1 By Quadrants

Referring to figure 5.22, suppose the coordinates of stations A and B are known to be (E_A, N_A) and (E_B, N_B) and that the whole-circle bearing of line AB (θ_{AB}) and the horizontal length of AB (D_{AB}) are to be calculated. The procedure is as follows.

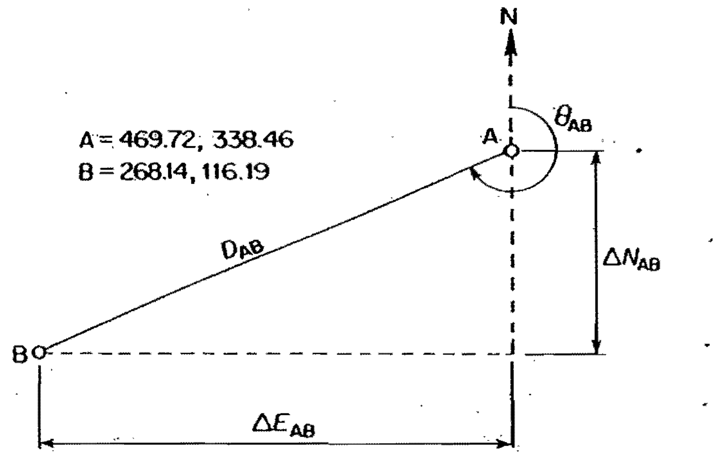


Figure 5.22

(1) A sketch showing the relative positions of the two stations should *always* be drawn in order to determine in which *quadrant* the line falls. This is most important as the greatest source of error in this type of calculation is wrong identification of quadrant. For whole circle bearings the quadrants are shown in figure 5.23.

(2) θ_{AB} is given by (in figure 5.22)

$$\theta_{AB} = \tan^{-1} (\Delta E_{AB}/\Delta N_{AB}) + 180° = \tan^{-1} [(E_B - E_A)/(N_B - N_A)] + 180°$$

$$= \tan^{-1} \left[\frac{268.14 - 469.72}{116.19 - 338.46}\right] + 180° = \tan^{-1} \left[\frac{-201.58}{-222.27}\right] + 180°$$

$= \tan^{-1}(0.906\,915) + 180^\circ$

$= 42^\circ\ 12'\ 19'' + 180^\circ$

Hence

$\theta_{AB} = 222^\circ\ 12'\ 19''$

It must be realised that, *in general*, the final value of θ_{AB} will depend on the quadrant of the line and a set of rules, based on the quadrant in which the line falls,

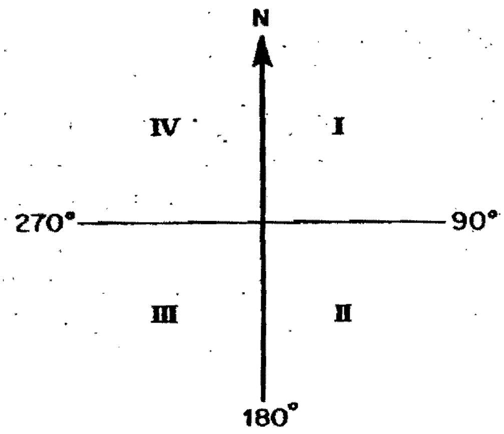


Figure 5.23 *Quadrant*

can be proposed to determine the whole circle bearing. These rules are shown in table 5.4.

(3) Having now found θ_{AB}, D_{AB} is given by

$$D_{AB} = (\Delta E_{AB}/\sin\theta_{AB}) = (\Delta N_{AB}/\cos\theta_{AB})$$

For figure 5.22

$$D_{AB} = \frac{-201.58}{\sin 222^\circ\ 12'\ 19''} = \frac{-201.58}{-0.671\,789} = 300.06\text{ m}$$

$$= \frac{-222.27}{\cos 222^\circ\ 12'\ 19''} = \frac{-222.27}{-0.740\,743} = 300.06\text{ m} \quad \text{(check)}$$

When evaluating D, *both* of the above should be calculated as a check against gross error. In the case where *small* differences occur between the two results, the correct answer is given by the trigonometric function which is the slower changing,

TABLE 5.4

QUADRANT	I	II/III	IV
FORMULA	$\theta = \tan^{-1}(\Delta E/\Delta N)$	$\theta = \tan^{-1}(\Delta E/\Delta N) + 180^\circ$	$\theta = \tan^{-1}(\Delta E/\Delta N) + 360^\circ$
Note:	$(\Delta E/\Delta N)$ must be calculated allowing for their signs. For a line XY, $\Delta E_{xy} = E_y - E_x$ and $\Delta N_{xy} = N_y - N_x$		

for example, if $\theta = 5°$, D found from $(\Delta N/\cos\theta)$ gives the more accurate answer since the cosine function is changing less rapidly than the sine function at this angle value.

Alternatively, D may be given by $D = (\Delta E^2 + \Delta N^2)^{\frac{1}{2}}$. If an electronic calculator is being used then this method will give a satisfactory answer but provides no check on the result. For this reason, the method involving the trigonometrical functions is preferred.

Examples of this type of calculation are given in section 5.11 and section 5.12.2.

5.10.2 By Rectangular/Polar Conversion

If a calculator is available which is fitted with a rectangular/polar key, values of D and θ can be obtained directly. When using this function, the coordinate values must be entered into the calculator in the correct sequence otherwise the wrong bearing will be obtained. In all cases, if θ is displayed as having a negative value, 360° must be added to give the correct whole-circle bearing. This is due to the fact that calculators display θ either between 0° and +180° or between 0° and −180°.

5.11 The National Grid

All Ordnance Survey (OS) maps and plans in Great Britain are based on a rectangular coordinate system known as the *National Grid*. Whole-circle bearings on this co-ordinate system are related to the north axis of the National Grid, a direction known as *grid north*. The network of stations which form the basis of this grid are known as *triangulation stations* (see section 6.3.2) and these are specially constructed pillars or permanent marks on prominent features such as churches, tall buildings, water towers and so on. Triangulation stations, which are distributed throughout the country, have been very accurately coordinated on the National Grid by the OS and are, therefore, very useful in many engineering projects, particularly road schemes, when used as reference or control points for setting out works (see section 14.6) or as known points in link traverses.

The National Grid is derived from a *map projection*. This is a means of representing the curved surface of the Earth on a plane so that maps and grids can be drawn. In forming the National Grid, the relative positions of points on the grid are altered slightly from their ground positions as a result of using a map projection to account for the curvature of the Earth. Therefore, distances and bearings calculated from National Grid coordinates will not, in some cases, agree with their equivalent measured in the field.

To convert measured distances to grid distances the *scale factor* (F) must be used as follows

$$\text{grid distance} = \text{measured distance} \times F$$

The scale factor varies across the country, as shown in table 5.5.

In practice, bearings derived from National Grid coordinates are assumed to agree with those obtained from measurements, provided the length of any individual line is less than 10 km.

TABLE 5.5

National Grid Easting (km)		Scale Factor (F)
400	400	0.999 60
410	390	60
420	380	61
430	370	61
440	360	62
450	350	63
460	340	65
470	330	66
480	320	68
490	310	70
500	300	72
510	290	75
520	280	78
530	270	81
540	260	84
550	250	88
560	240	92
570	230	0.999 96
580	220	1.000 00
590	210	04
600	200	09
610	190	14
620	180	20
630	170	25
640	160	31
650	150	1.000 37

5.12.2 Link Traverse

Question

A link traverse was run between stations A and X as shown in the traverse diagram of figure 5.25.

The coordinates of the controlling stations at the ends of the traverse are as follows

	E (m)	N (m)
A	1769.15	2094.72
B	1057.28	2492.39
X	2334.71	1747.32
Y	2995.85	1616.18

Calculate the coordinates of stations 1, 2, 3 and 4, adjusting any misclosure by the Transit method.

Solution

The complete solution is given in the traverse table shown in table 5.7.

(1) The solution follows the direction A to X as this will give the left-hand angles, as shown in figure 5.25.

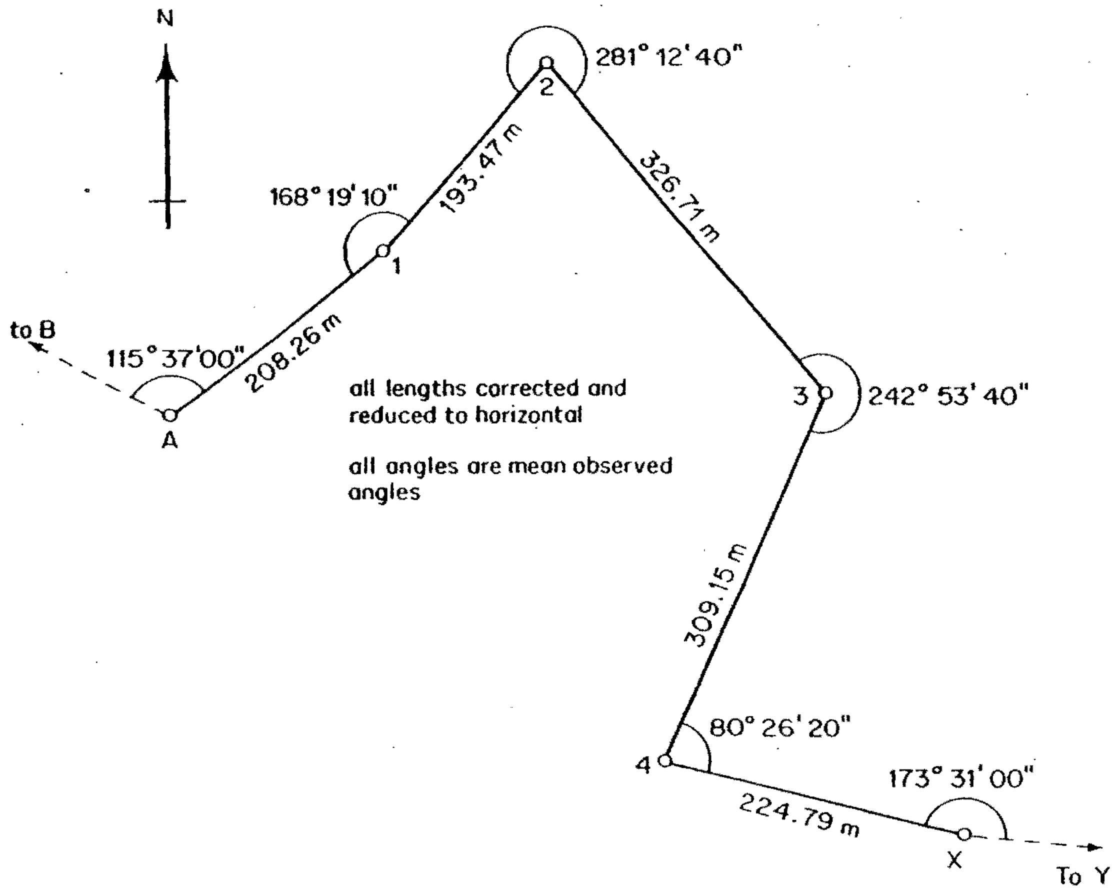


Figure 5.25

(2) When link traversing, the starting and closing bearings may either be given directly or implied by the coordinates of the stations used to start and end the traverse. In this case, coordinates are given and it is necessary to compute the initial and final bearings.

(a) *Initial back bearing AB*

Figure 5.25 is a sketch of the traverse, approximately to scale, and, therefore, shows that the bearing AB is in the *fourth* quadrant. Hence, the whole-circle bearing, θ_{AB}, is given by (see section 5.10.1)

$$\begin{aligned}\theta_{AB} &= \tan^{-1} (\Delta E_{AB}/\Delta N_{AB}) + 360^\circ \\ &= \tan^{-1} [(1057.28 - 1769.15)/(2492.39 - 2094.72)] + 360^\circ \\ &= \tan^{-1} (-711.87/397.67) + 360^\circ \\ &= \tan^{-1} (1.790\,10) + 360^\circ\end{aligned}$$

Hence

$$\theta_{AB} = -60^\circ 48'40'' + 360^\circ$$

Therefore

$$\theta_{AB} = 299^\circ 11'20''$$

Alternatively, a rectangular/polar conversion can be used as described in section 5.10.2.

(b) *Final forward bearing XY*

From figure 5.25, the bearing XY lies in the *second* quadrant hence

$$\begin{aligned}\theta_{XY} &= \tan^{-1} (\Delta E_{XY}/\Delta N_{XY}) + 180^\circ \\ &= \tan^{-1} [(2995.85 - 2334.71)/(1616.18 - 1747.32)] + 180^\circ \\ &= \tan^{-1} (661.14/-131.14) + 180^\circ \\ &= \tan^{-1} (-5.041\,48) + 180^\circ\end{aligned}$$

Hence

$$\theta_{XY} = -78^\circ 46'50'' + 180^\circ$$

Therefore

$$\theta_{XY} = 101^\circ 13'10''$$

Again, a rectangular/polar conversion can also be used.

(3) The angular misclosure is found as follows (see also section 5.8.2)

$$\text{Sum of left-hand angles} = 1061^\circ 59'50''$$

$$\begin{aligned}&(\text{final forward bearing} - \text{initial back bearing}) + (n-1) \times 180^\circ \\ &= (101^\circ 13'10'' - 299^\circ 11'20'') + (5 \times 180^\circ) \\ &= (461^\circ 13'10'' - 299^\circ 11'20'') + 900^\circ \\ &= 1062^\circ 01'50''\end{aligned}$$

The misclosure is, therefore, $-02'00''$ and each left-hand angle is adjusted by adding $20''$ to it.

TABLE 5.7

LINE / STATION / LINE	BACK BEARING / ADJUSTED LEFT HAND ANGLE / FORWARD BEARING			WHOLE CIRCLE BEARING θ			HORIZONTAL DISTANCE D	COORDINATE DIFFERENCES: CALCULATED ΔE	ΔN	ADJUSTMENTS δE	δN	ADJUSTED ΔE	ΔN	COORDINATES E	N	STATION
AB	299	11	20											1769.15	2094.72	A
A	115	37	20													
A1	54	48	40	54	48	40	208.26	+170.20	+120.01	+0.02	+0.01	+170.22	+120.02	1939.37	2214.74	1
1A	234	48	40													
1	168	19	30													
12	43	08	10	43	08	10	193.47	+132.28	+141.18	+0.02	+0.01	+132.30	+141.19	2071.67	2355.93	2
21	223	08	10													
2	281	13	00													
23	144	21	10	144	21	10	326.71	+190.40	-265.49	+0.02	+0.02	+190.42	-265.47	2262.09	2090.46	3
32	324	21	10													
3	242	54	00													
34	207	15	10	207	15	10	309.15	-141.57	-274.83	+0.02	+0.02	-141.55	-274.81	2120.54	1815.65	4
43	27	15	10													
4	80	26	40													
4X	107	41	50	107	41	50	224.79	+214.15	-68.33	+0.02	+0.00	+214.17	-68.33	2334.71	1747.32	X
X4	287	41	50													
X	173	31	20													
XY	101	13	10	101	13	10										
							Σ 1262	Σ+565.46	Σ-347.46	Σ+0.10	Σ+0.06	Σ+565.56	Σ-347.40			

ACTUAL SUM OF LEFT HAND ANGLES = 1061°59'50"

REQUIRED SUM OF LEFT HAND ANGLES = 1062°01'50"

MISCLOSURE = -02'00"

ADJUSTMENT TO EACH OBSERVED ANGLE = +20"

$\Sigma\Delta E$ = +565.46 $\quad$ $\Sigma\Delta N$ = -347.46 $\quad$ abs $\Sigma\Delta E$ = 849

$E_X - E_A$ = +565.56 $\quad$ $N_X - N_A$ = -347.40 $\quad$ abs $\Sigma\Delta N$ = 870

e_E = -0.10 $\quad$ e_N = -0.06 $\quad$ ADJUSTMENT TO $\Delta E/\Delta N$ BY TRANSIT

$e = ((-0.10)^2 + (-0.06)^2)^{\frac{1}{2}} = 0.12$

FRACTIONAL LINEAR MISCLOSURE = 1 in 10 500

- an open traverse has the following data:

line	WCB	Length (m)
AB	60 20 30	125.745
BC	308 15 50	222.861
CD	172 47 00	53.221

-Given that the coordinates of station A are 100 east and 100 north, calculate the coordinates of stations B, C and D.

line	Length	WCB	E	N
AB	125.745	$60^\circ 20' 30''$	109.271	62.222
BC	222.861	$308^\circ 15' 50''$	-174.983	138.014
CD	53.221	$172^\circ 47' 00''$	6.685	-52.799

Pts	E(m)	N(m)
A	100	100
B	209.271	162.222
C	34.288	300.236
D	40.973	247.437

the following data were obtained in a short traverse ABCDA

line	Bearing	Length (m)
AB	W.C.B= $70^\circ\ 34'$	295.54
BC	Q.B.= S $9^\circ 18'$E	269.42
CD	W.C.B.= $210^\circ 30'$	184.32

Find the length and the W.C.B. of line DA, calculate also the coordinates of C if the coordinates of A are (2250 E,1720 S).

line	Length	WCB	ΔE	ΔN
AB	295.54	$70^\circ 34'$	278.70	98.33
BC	269.42	$170^\circ 42'$	43.54	-265.88
CD	184.32	$210^\circ 30'$	-93.55	-158.81
DA	?	?	X	Y

Closed Travese : Sum of ΔE =0 and Sum of ΔN =0

DA= $\sqrt{X^2 + Y^2}$ = 398.509 m

Bearing of DA = $\tan^{-1}(\Delta E/\Delta N)$= N $35^\circ\ 01'\ 12''$ w
C (2572.24 E, 1887.5 S)

Engineering Leveling

BASICS OF LEVELLING

USES OF LEVELLING

In the context of tidal measurements, levelling is used for the following purposes:

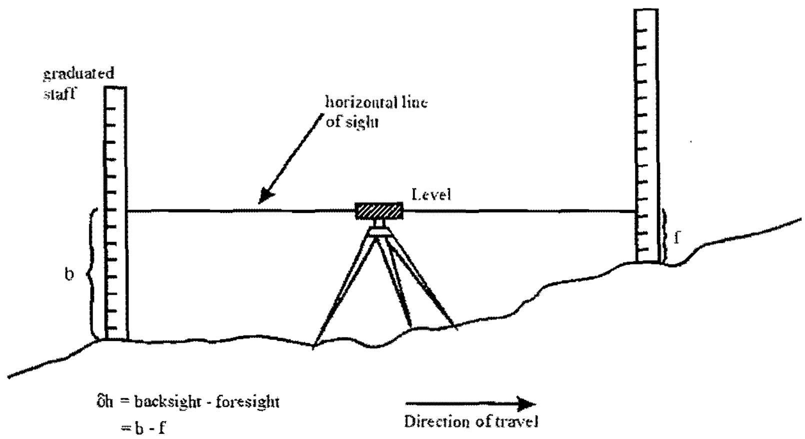
Referencing of Tide Gauges: To determine and check the vertical stability of the tide gauge bench mark (TGBM) with respect to reference points (benchmarks) in its immediate vicinity. In order to isolate any local movements, there should be at least three such benchmarks, and the levelling should be repeated on an annual or semi-annual basis.

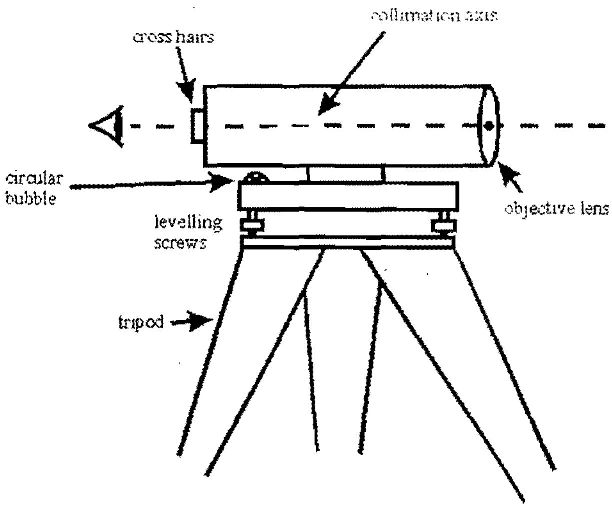
Connection to GPS Reference Points: To determine its regional stability and to separate sea level rise from vertical crustal motion, the TGBM should be connected via GPS to reference stations fixed in a global co-ordinate system. Generally speaking, the GPS antenna cannot be directly placed on the TGBM and a GPS reference point must be established a short distance away. This must be connected to the TGBM by levelling.

Connection to National Levelling Network: Mean sea level is used to define vertical datums for national surveying and mapping - hence the TGBM must be connected to the national levelling network. Connection to the network will also allow all tide gauges to be connected to each other, providing information on spatial variations in mean sea level.

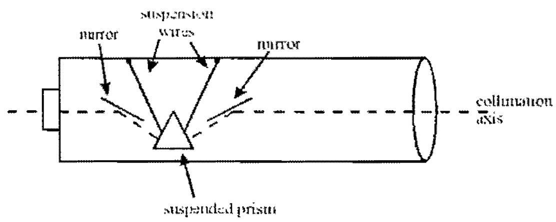
PRINCIPLE OF DIFFERENTIAL LEVELLING

Differential levelling provides a means of accurately measuring height differences between points some tens of metres apart. A level is set up on a tripod and levelled so that the line of sight is horizontal:

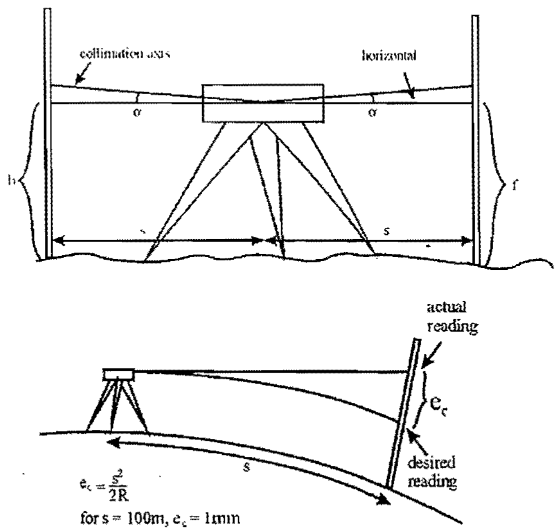




A schematic illustration of a basic level



Automatic Compensator:



Collimation Error:

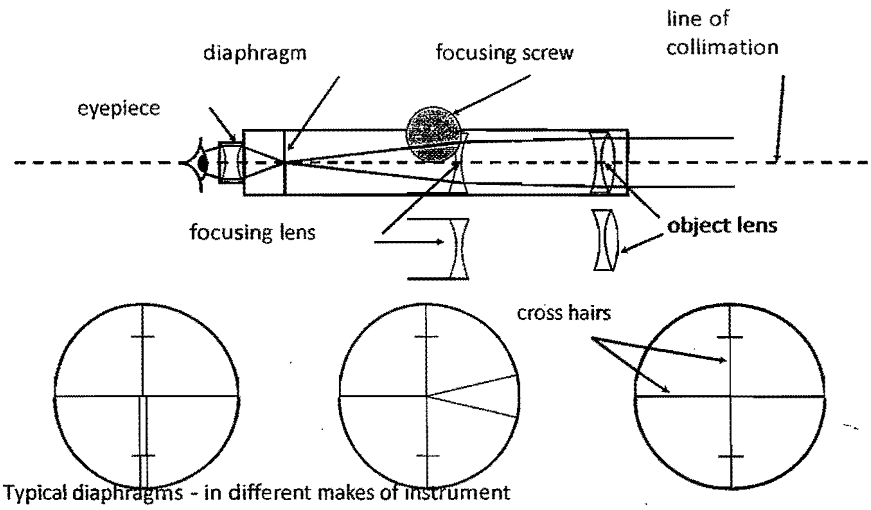
Vertical Distances - Levelling

Measuring the height

Measuring and calculating the height of a point relative to another point

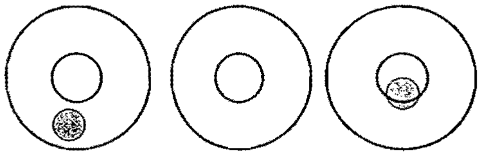
Level   Spirit level   Water level   **Optical** level

A surveying optical telescope



Typical diaphragms - in different makes of instrument

Pond Bubble

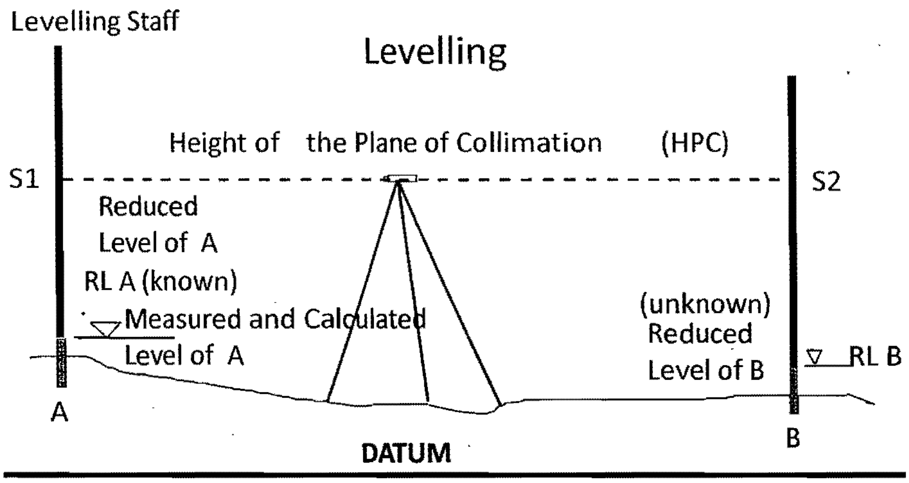


When pond bubble is centred the instrument's standing axis is approximately vertical.

The compensators in the instrument take over and adjust the optical Line of Collimation so that it is horizontal (hopefully)

When the instrument is rotated the compensators ensure that a **horizontal plane of collimation** is swept out (hopefully)

Levelling

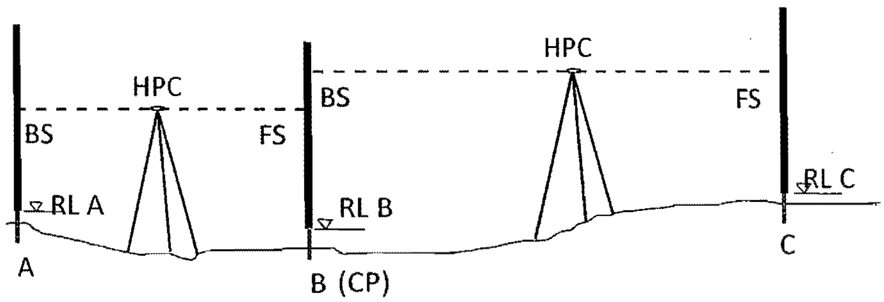


HPC = RL A + S1

RL B = HPC - S2

•Definitions

• **Bench Mark** " BM": it is a point with known level, given from municipality measured as AOD level
• **backsigth** "BS": it is the first staff reading
•**Intermediate sigth** "IS": it is the intermediate readings of staff in which the position for the level did not change
•**Foresight** "FS": it is the last reading of staff before changing the position for the level
• **turning or changing point** "CP or TP": it is a common point between the positions of the two level , i.e. the following reading will be B.S and F.S for the new position



RL A is known

HPC = RL A + BS

RL B = HPC - FS

Now the RL B is known

So we can repeat the process

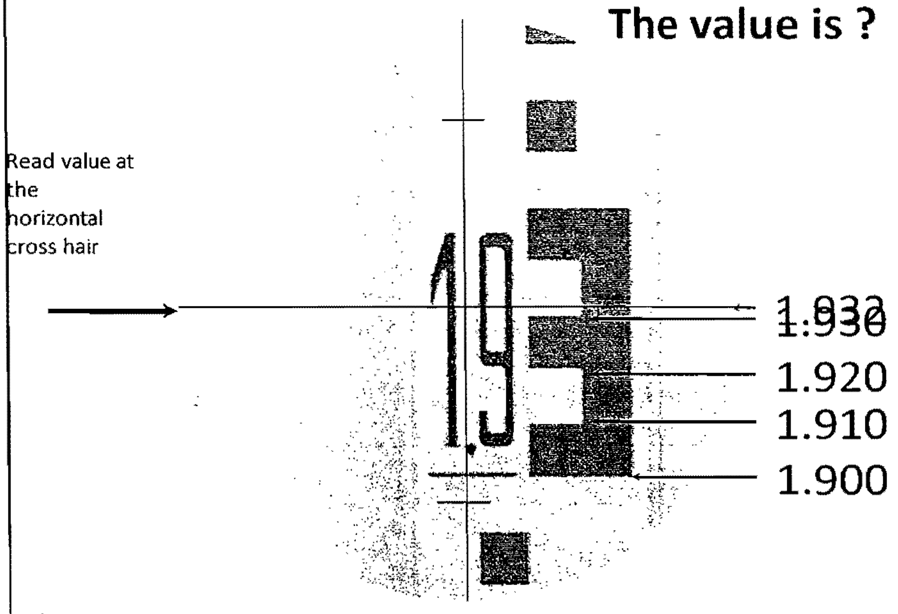
HPC = RL B + BS

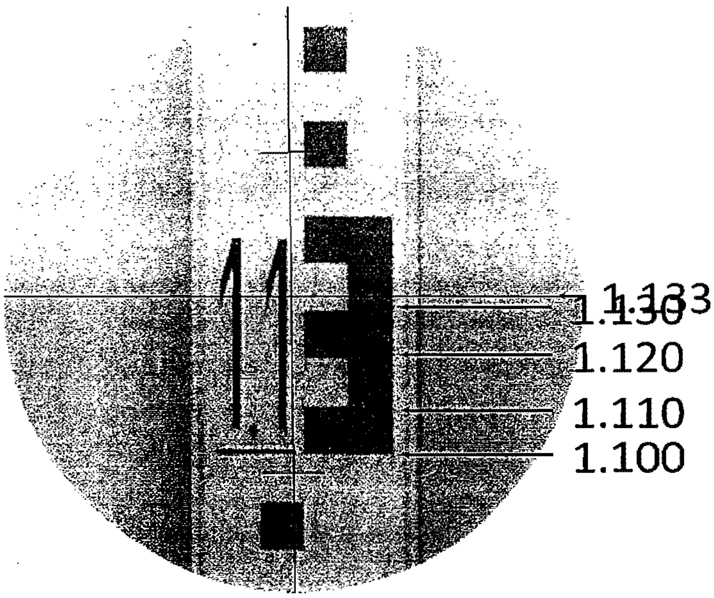
RL C = HPC - FS

Generally : **HPC = Known RL + Back Sight**

Unknown RL = HPC - Fore Sight

Reading an E-type levelling staff



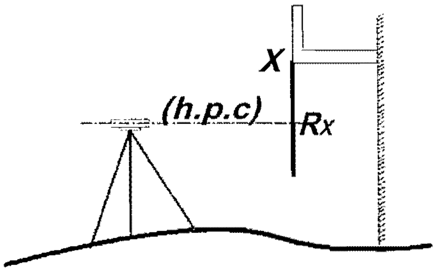


Topographic Surveying

Case of inverted sight:

$$X = (h.p.c) - (-R_X)$$

$$X = (h.p.c) + R_X$$



Put the reading in a negative sign

Topographic Surveying

4. Reciprocal Leveling

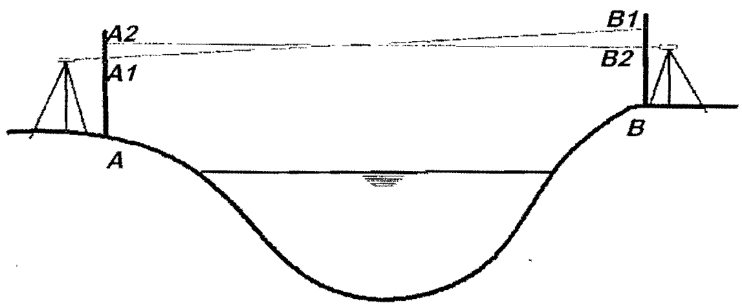
The purpose of this type of leveling is to get the difference in level between Two points lie in opposite of lake, a river or a big hall the following procedures have to be followed.

1. **Use two instruments**
2. **Take the readings at the same time**
3. **Exchange the two instruments.**

We will have four difference and we will take the average

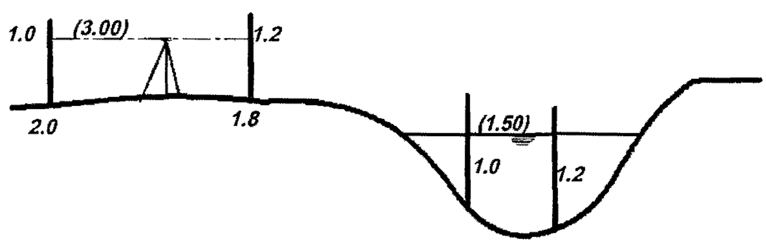
Topographic Surveying

$$\Delta h = \frac{[(A_1 - B_1) + (A_2 - B_2) + (A_3 - B_3) + (A_4 - B_4)]}{4}$$



Topographic Surveying

Case of water body:



(a) = 1.50 - 1.00 = 0.50m

(b) = 1.50 - 1.20 = 0.30m

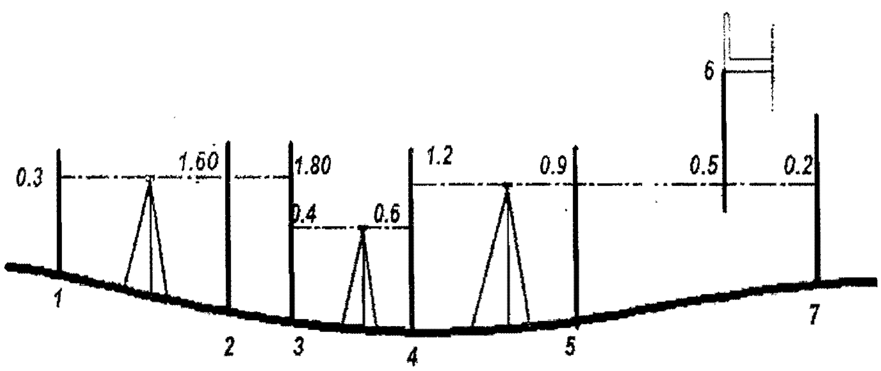
Topographic Surveying

5. Height of Collimation Method:

The following readings were taken by a level:
0.30 – 1.60 – 1.80 – 0.40 – 0.60 – 1.20 – 0.90- 0.50 – 0.20.
And:
1. The forth and sixth readings are back sights.
2. The third and fifth readings are forsights.
3. The third and forth points are turning points.
4. The level was moved after the third and the fifth readings.

The level of point number 1 = 2.10 m and the reading number 8 is Inverted sight.

Topographic Surveying



Topographic Surveying

Point	B.S	I.S	F.S	H.P.C	levels	Dist.	Rem.
1	0.30			2.40	2.10	000	known
2		1.60			0.80	100	
3	0.40		1.80	1.00	0.60	200	Turning
4	1.20		0.60	1.60	0.40	300	Turning
5		0.90			0.70	400	
6		-0.50			2.10	500	Inverted
7			0.20		1.40	600	
$\sum$	1.90	2.00	2.60		6.00		

Topographic Surveying

Mathematical checks:

1. # of b.s readings = # of f.s readings
   3   3
   ok
2. $\sum$ b.s - $\sum$ f.s = (last point) - (first point)
   1.90 – 2.60 = 1.40 - 2.10
   -0.70 = -0.70
   ok
3. $\sum$ Levels except the first point + $\sum$ I.S + $\sum$ F.S + =

   $\sum$ (H.P.C) * # of times to get new levels

Left side = 6.00 + 2.00 + 2.60 = 10.60

Right side = 2.40 * 2 + 1.00*1+ 1.6*3 =10.60 = left side

Ok

Topographic Surveying

The allowable error in leveling is $n\sqrt{k}$ mm

As n = 5, 10, or 12

K is the length of leveling line in km

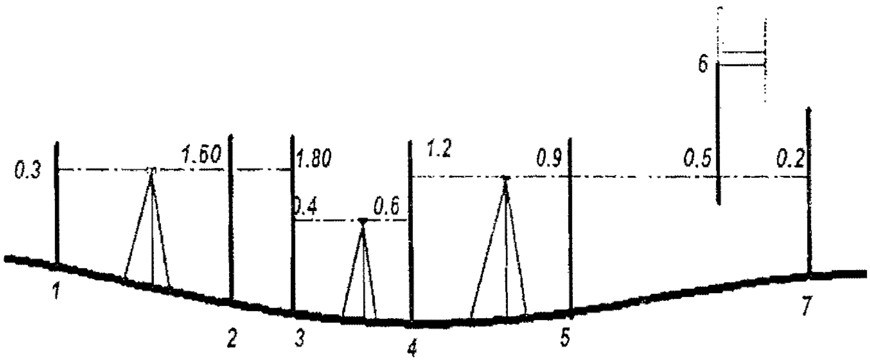
Topographic Surveying

7. Rise and fall Method:

The following readings were taken by a level:
0.30 – 1.60 – 1.80 – 0.40 – 0.60 – 1.20 – 0.90- 0.50 – 0.20.
And:
1. The forth and sixth readings are back sights.
2. The third and fifth readings are forsights.
3. The third and forth points are turning points.
4. The level was moved after the third and the fifth readings.

The level of point number 1 = 2.10 m and the reading number 8 is Inverted sight.

Topographic Surveying



Topographic Surveying

Point	B.S	I.S	F.S	Rise	Fall	levels	Dist.	Rem.
1	0.30					2.10	000	known
2		1.60			1.30	0.80	100	
3	0.40		1.80		0.20	0.60	200	Turning
4	1.20		0.60		0.20	0.40	300	Turning
5		0.90		0.30		0.70	400	
6		-0.50		1.40		2.10	500	Inverted
7			0.20		0.70	1.40	600	
∑	1.90	2.00	2.60	1.70	2.40	6.00		

Topographic Surveying

Mathematical checks:

1. # of b.s readings = # of f.s readings

   3 3

   ok

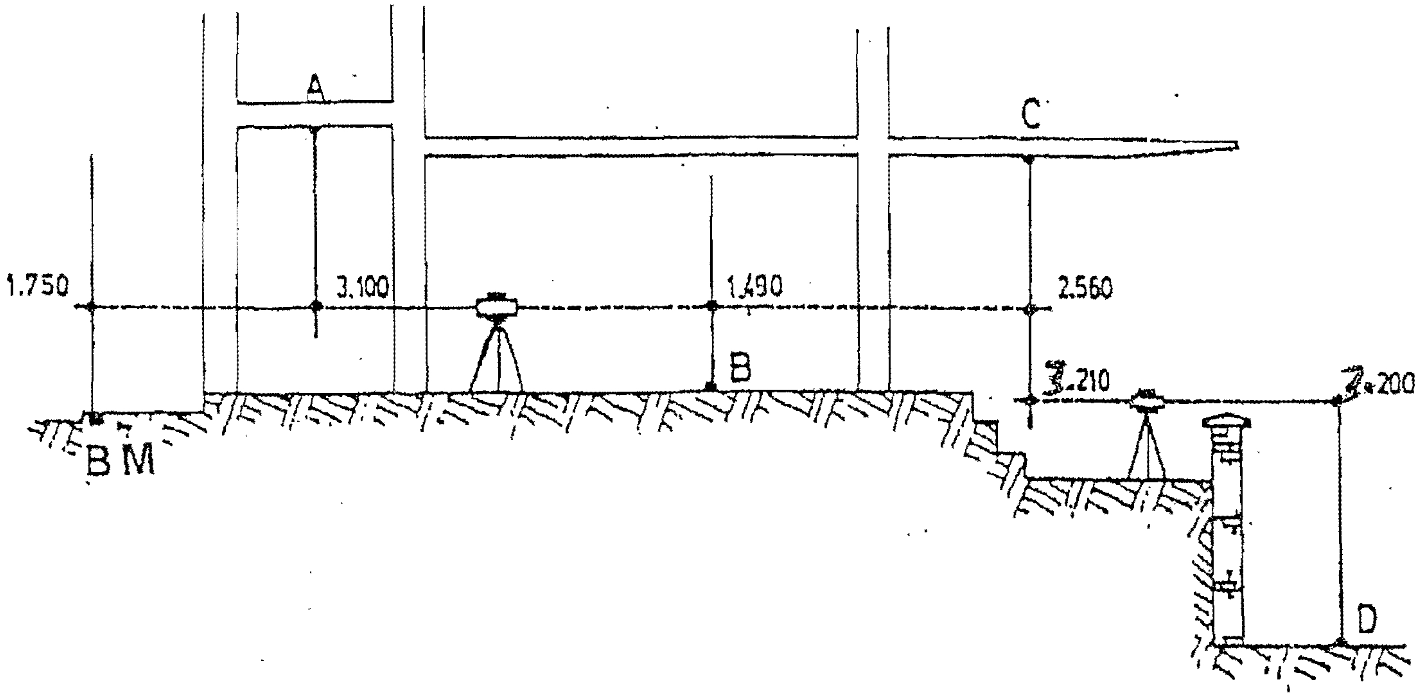
2. $\sum$ b.s - $\sum$ f.s = (last point) - (first point) = $\sum$ rise - $\sum$ fall

   1.90 – 2.60 = 1.40 - 2.10 = 1.70 - 2.40

   -0.70 = -0.70 = -0.70

   ok

figure (1) shows the readings observed to points A,B,C and D on the multi-storey buildings. Given that the reduced level of the BM is **(72.30) m** above datum, calculate the reduced levels of all points by HPC method.



Fig(1)

Pts	Staff Readings			HPC	Rls	Remarks
	B.S	I.S	F.S			
BM	1.750			74.05	(72.30)	Bench Mark
A		-3.100			(77.15)	inverted
B		1.490			(72.56)	
C	-3.210		-2.560	73.40	(76.61)	cp
D			3.200		(70.20)	

- The following readings were taken with a metric staff on a series of pegs at 100 intervals along the line of proposed trench:

  2.15, 2.85,3.51,(1.80),1.58,2.24,2.94,(1.68),2.77,3.06,3.83

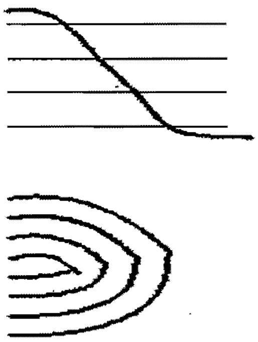
  The first reading was taken on BM (30.75) m AOD outside the line and the last reading was taken on BM (29.94) m. the other readings were taken on the heads of the pegs, where the readings between brackets are FS readings .you are required :

  1) Calculate the reduced level trench using HPC method and carry out the normal checks.
  2) Draw a longitudinal section showing the ground and formation levels, then compute the height of cut or fill at pegs if the trench is to be excavated from the first peg at formation level (28.50)m AOD and falling to the last peg at gradient of 1: 250.

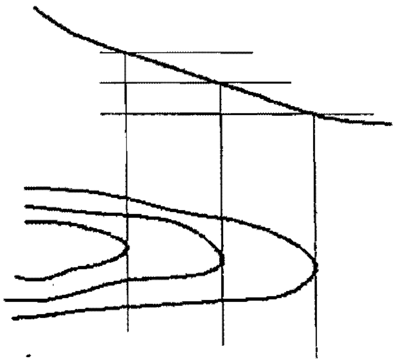
BS	IS	*FS*	*HPC*	*Surfacer reduced level*	*Grade Reduced Level*	*Fill*	*Cut*	*Remarks*
2.15			32.90	(30.75)	---------		------	BM1
	2.85			(30.05)	(28.50)		1.55	0
	3.51			(29.39)	(28.10)		1.29	100 m
1.58		1.80	32.68	(31.10)	(27.70)		3.4	200 m
	2.24			(30.44)	(27.30)		3.14	300 m
	2.94			(29.74)	(26.90)		2.84	400 m
2.77		1.68	33.77	(31.00)	(26.50)		4.50	500 m
	3.06			(30.71)	(26.10)		4.61	600 m
		3.83		(29.94)	----------		------	BM2

Contouring

Topographic Surveying

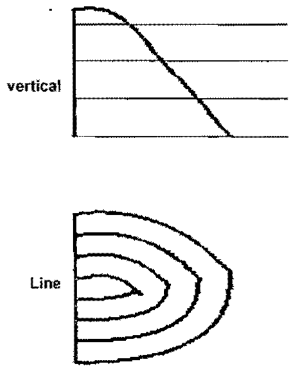


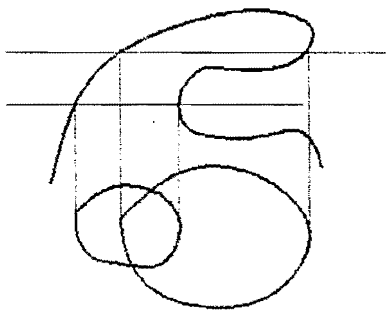
steep slope



Mild slope

Topographic Surveying

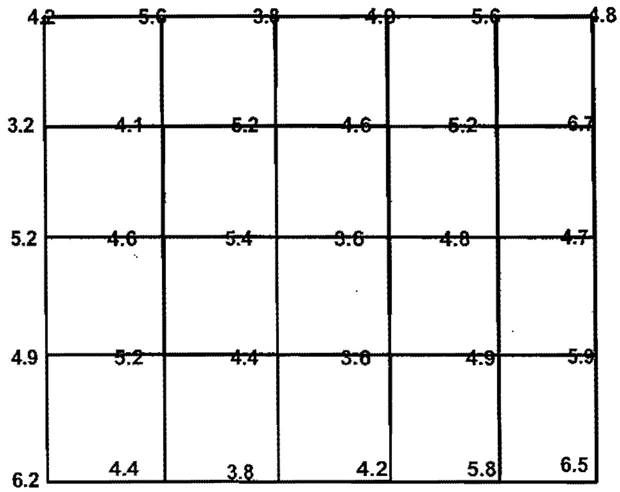




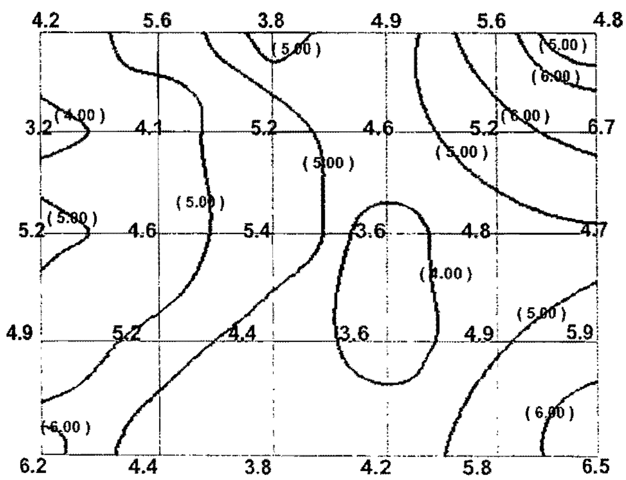
حالة الكهف

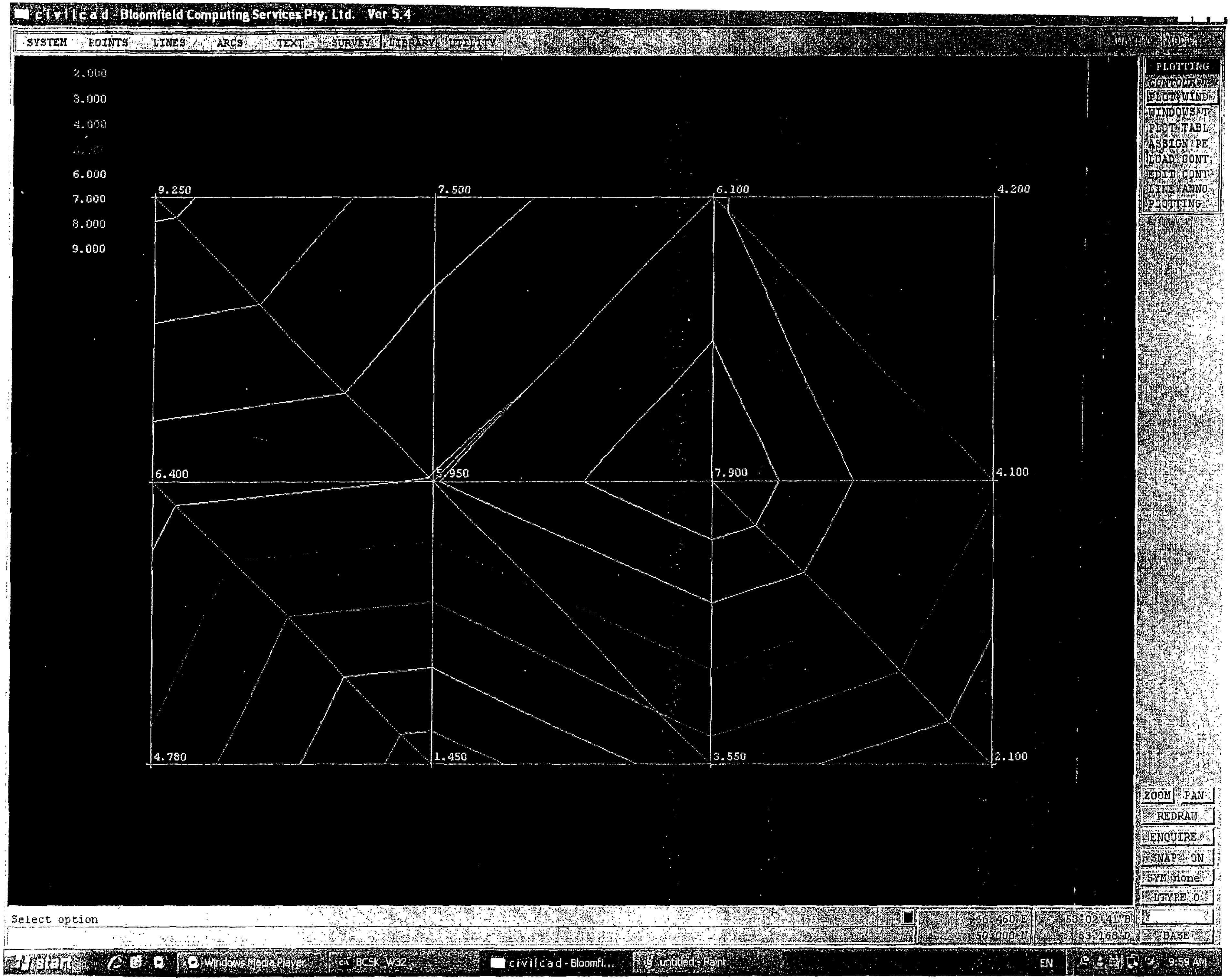
Topographic Surveying

Example (4) We have below spot levels for an area and it is required to draw the contour lines for an interval.



Topographic Surveying




civilcad Bloomfield Computing Services Pty. Ltd. Ver 5.4
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5.950
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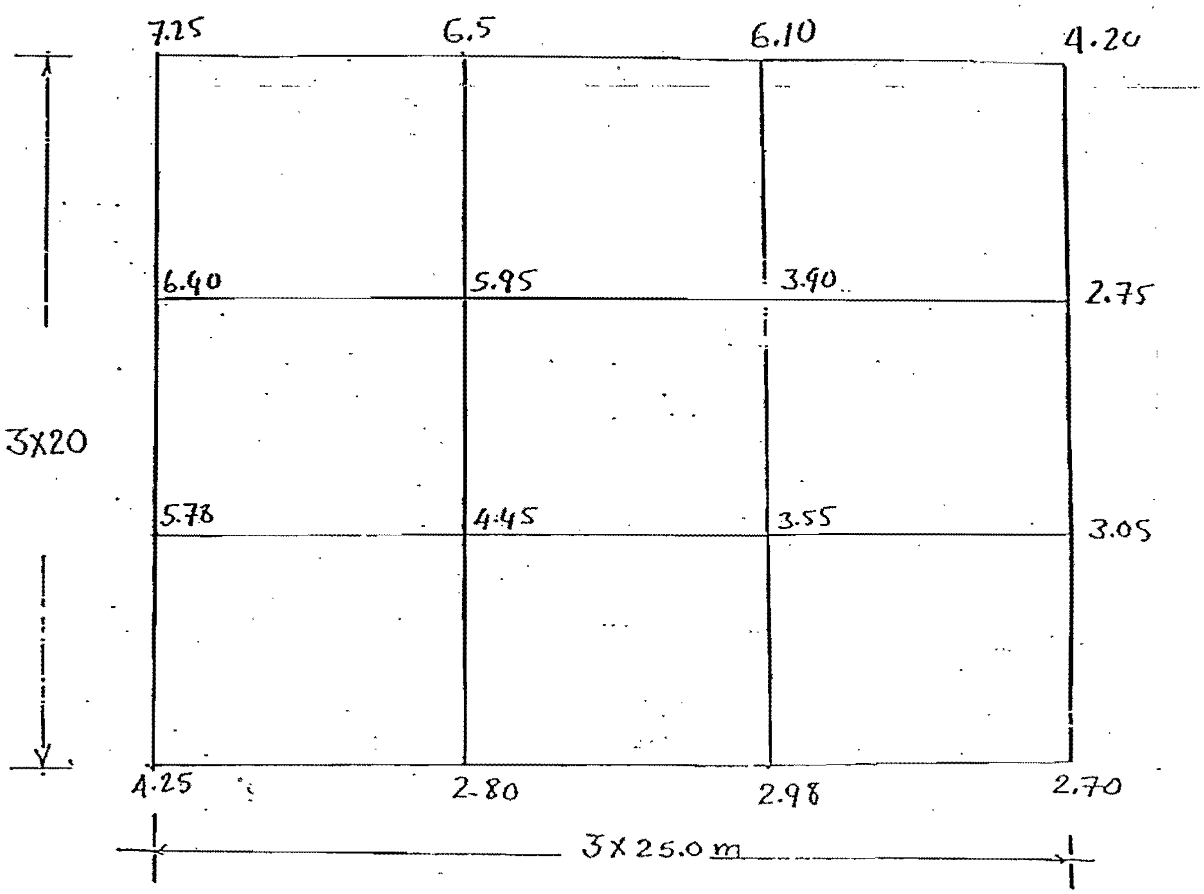
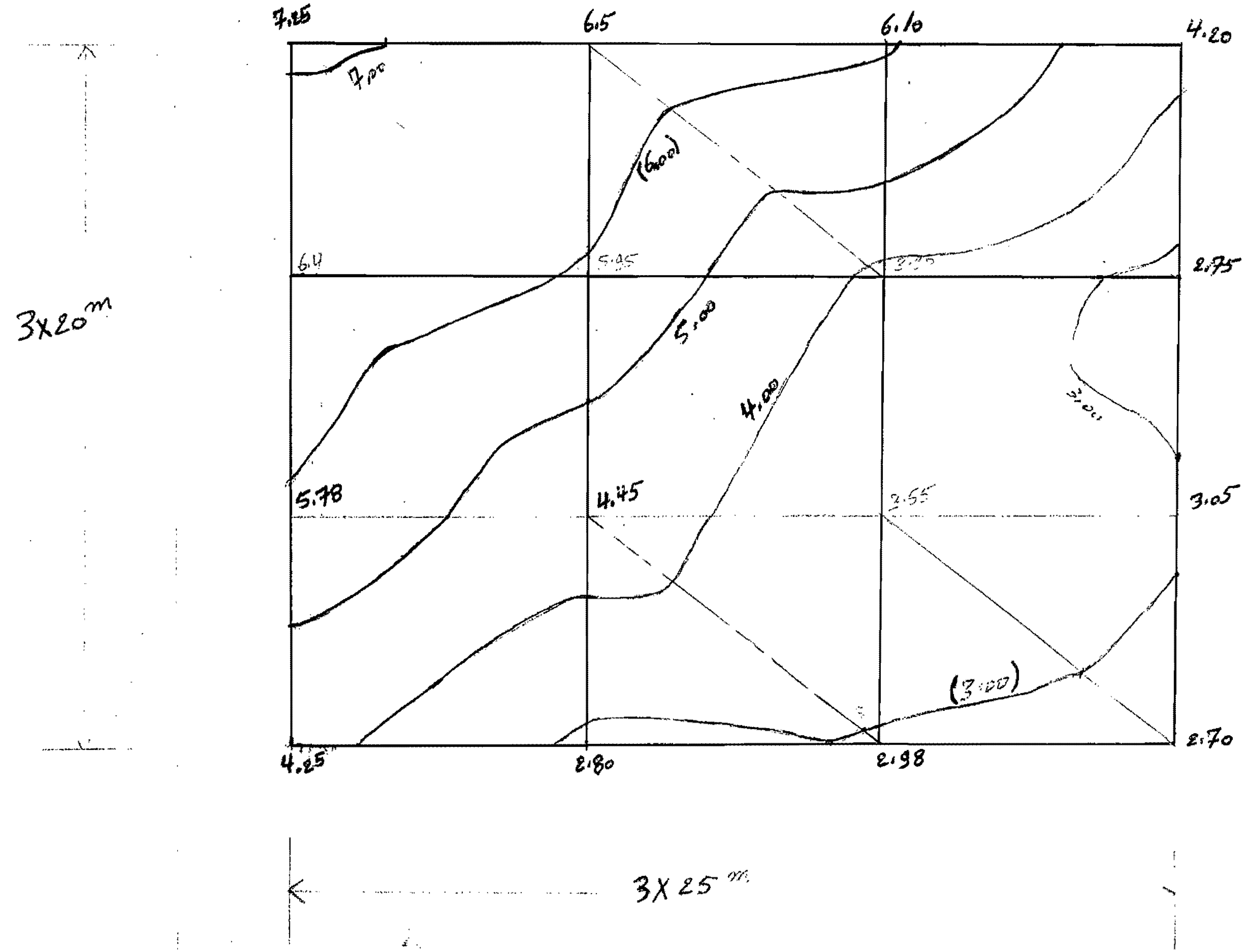
- Draw to scale the contour lines of the following leveling net if the vertical interval = 1: 500





7.25
6.5
6.1o
4.20
7.00
(6.00)
6.4
5.95
3.75
2.75
5.00
4.00
3.00
5.78
4.45
2.55
3.05
(3.00)
4.25
2.80
2.98
2.70
3x20 m
3X 25 m

Scale; 1/500

Problems

Beirut Arab University
Civil Engineering Dept.

Engineering Surveying
Exercise No. 1

LINEAR MEASUREMENTS

PART I - Scales

1- Calculate the scale of plan where 1 cm represents 22.50 m.

2- It is required to draw a linear scale 1:5000 to read up to 25m (least count).

3- Draw a scale for a map scaled by 1:2500 to read 10m, and find out the length 275m.

4- The scale of a map is 1:32000, it is required to draw a linear scale to read 1 km.

5- Calculate the true Area if it is measured on map drawn by scale 1/500 with 7 cm^2.

PART II – Errors in Linear Measurements

1- A survey line AB was measured along a 6 degree gradient. The slope distance was 49.75m. Calculate the plan length of the line. (49.48)

2- A survey line XY is measured along a steep gradient of 8^0 by two survey teams A and B. Team A used a 30 m shrunk by 50 mm, the measured length was 85.24 m. Team B used a 20 m steel tape which was actually 20 mm too long. The measured length was 84.94 m.
Calculate the most probable horizontal corrected length XY (84.23).

3- A line AB known to be 65.35 m long was checked using a 20 m tape. It was found to measure 65.10 m. Calculate the actual length of the tape (20.077).

4- Calculate the corrected plan length of line AD measured in 3 sections as follows:

AB= 102.3 ms angle of slope= $6^\circ 30'$

BC= 37.93 ms. Difference in level=5.88 ms

CD=48.2 ms gradient = $-5^\circ 00'$

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Civil Engineering Dept.

Engineering Surveying
Exercise No. 2

Compass Surveying

1)- ***for the whole circle bearings (WCB) : $75^\circ, 130^\circ, 240^\circ, 310^\circ$***
Determine the Reduced Bearing (RB). (use a neat sketch) .

2)- ***Determine the WCB for the following RB :***
a- N $18^\circ 20'$ E   b- S 46° 24 E$'$ c- S $25^\circ 47'$ W

3)- ***The magnetic bearing of a line is observed to be $120^\circ 25'$. What is The true bearing if the magnetic declination is 23°W ?***

4)- ***Explain ,by means of diagrams and notes, the difference between :***
a)-Whole-circle bearing and quadrant bearing .
b)- Forward and back bearing.

5)- ***The geographic azimuth of a line AB was found to be $346^\circ 20'$. At a certain time of the day the magnetic bearing of the same line was : $03^\circ 23'$. Another line CD was magnetically observed as $195^\circ 20'$***
Calculate :
a)- ***The magnetic declination .***
b)- ***true bearing of line CD.***

6)- ***For the bearings of line AB complete the following :***

Magnetic bearing	***60°***	***230°***	***.....***
True bearing	***.....***	***221°***	***166°***
Angle of declination	***10° W***	***.....***	***4° W***

7)- ***At a given place the magnetic bearings of two lines are N $48^\circ 32'$ E And S $54^\circ 36'$E . If the magnetic declination is $2^\circ 50'$E , What are the True bearings of the lines .***

8)- ***Change the following true bearings to magnetic bearings for $04^\circ 30'$ W magnetic declination: N $2^\circ 06'$ E ,N $4^\circ 10'$ W and S $87^\circ 39'$E.***

9)- *In 1860 the magnetic bearing of a line was S 86°30′W and the Magnetic declination was 4°30′ W. Compute the magnetic bearing Of this line at this year if the magnetic declination is now 2°E. What is the true bearing at this year ?*

10)- *The following readings are measured for the traverse ABCD. Adjust the readings:*

LINE	*FOR BEARING* ° ′	*BACK BEARING* ° ′
AB	*42° 09′*	*225° 18′*
BC	*105° 28′*	*284° 48′*
CD	*209 02*	*28 50*
DE	*267 58*	*86 18*
EA	*316 10*	*135 54*

11)-Correct the following observation for the local attraction

Line	F.W.C.B	B.W.C.B
AB	118^0 20′	300^0 40′
BC	224^0 40′	46^0 10′
CA	39^0 10′	219^0 50′

Calculate the corrected R.B.B for the line CA.

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Engineering Surveying
Exercise No. 3

Theodolite surveying

1- The following angles were measured on a closed theodolite traverse:

Instrument Station	Target Station	Face Left
A	D	00° 10′ 20″
	B	89 26 40
B	A	89 26 40
	C	189 05 20
C	B	189 05 20
	D	269 29 40
D	C	269 29 40
	A	00 09 00

Calculate:

a) The measured angles,
b) The angular correction,
c) The corrected angles,
d) The quadrant bearing of each line given that bearing AD is 45°36′ 00″

2- Calculate the closing error and accuracy of the following traverse:

Line	*Length (m)*	*Bearing*
AB	110.20	156° 40′ 00″
BC	145.31	75 18 00
CD	98.75	351 08 00
DE	163.20	276 29 00
EA	52.34	187 27 00

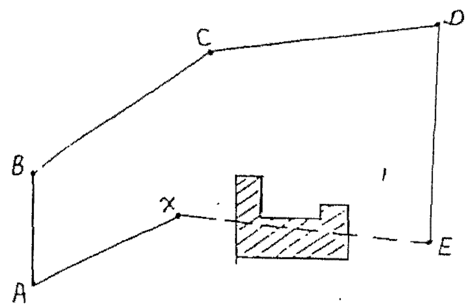
3- The partial coordinates of closed traverse are:

Line	*Length (m)*	*Difference in Easting*	*Difference in Northing*
PQ	252.41	0.00	252.41
QR	158.75	-110.76	-113.82
RS	153.50	-25.24	-151.41
SP	136.74	+136.15	12.67

Distribute the closing error by Bowditch's rule and calculate the corrected partial coordinates.

4) The following data refer to an open theodolite traverse round an old farm house which is to be demolished at a future date.

Line	*Plan Length (m)*	*Angle*	*Observed Value*
XA	160.00	XAB	330° 00′
AB	186.40	ABC	251 30
BC	234.00	BCD	198 30
CD	170.60	CDE	280 45
DE	138.00		



Given that the coordinates of station X are 100 ME and 100 mE and 100 mN and that the forward bearing of line XA is 210° 00′ 00″, calculate :

a) The total coordinate of each station,
b) The length of and the bearing of a proposed sewer that is to be laid between the points X and E.
c) The values of the angles to be set out at X and E by the theodolite to establish the line of the sewer.

5) The following lengths and angles were obtained on a closed theodolite traverse using a 20 m steel band and the theodolite:

Line	*Plan length*	*Quadrant Bearing*
AB	167.25	North
BC	228.34	N 30° 24′ 00 E
CD	367.50	S 18° 16 40 E
DA	220.70	N 89 28 40 W

It is suspected that there is a gross error in one of the linear measurements. Calculate the total coordinates of each station: hence determine the erroneous line and the probable reason for the wrong measurement.

6)- For the closed traverse given in fig.(1), it is required to:
a) calculate the length and the bearing of the Sid DA and the RB of AD.
b) Calculate the coordinates of stations B,C & D if the coordinate of A are (200 E, 200N).

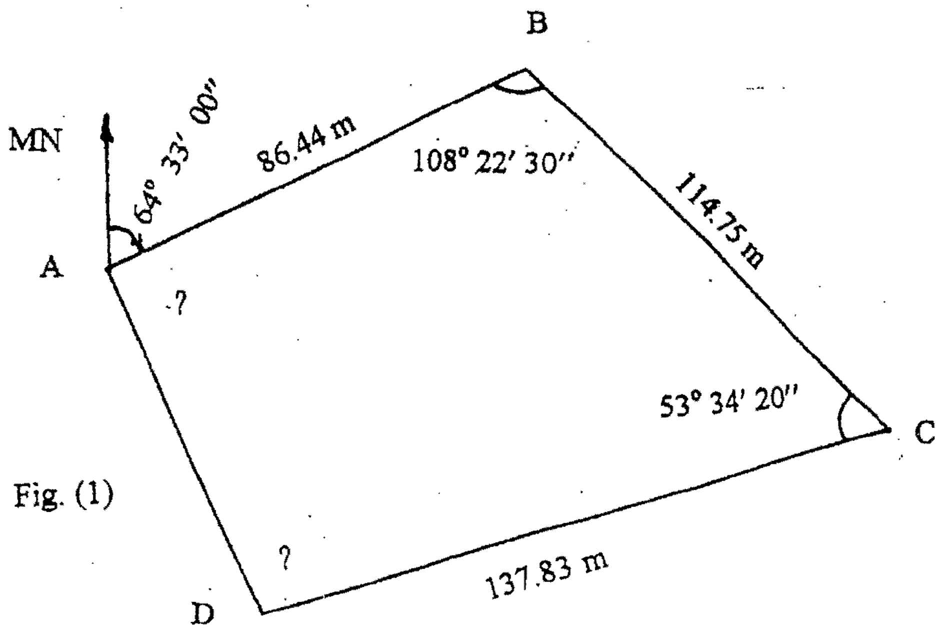


Fig. (1)

7) - Determine the corrected angles from the following observation taken about station X.

Observed target	F.R	F.L
A	176°04′ 00″	356°06′10″
B	318 04 40	137 56 10
C	95 12 10	275 14 20
A	176 02 00	356 03 20

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Engineering Surveying
Exercise No. 4

Theodolite surveying

1) A link traverse was run between stations A and X as shown in the traverse diagram of Fig (1).

The coordinates of controlling stations at the ends of the traverse are as follows :

Stn.	E(m)	N(m)
A	1769.15	2094.72
B	1057.28	2492.39
X	2334.71	1747.32
Y	2995.85	1616.18

Calculate the coordinates of stations 1,2,3 and 4, adjusting any misclosure by the Transit method.

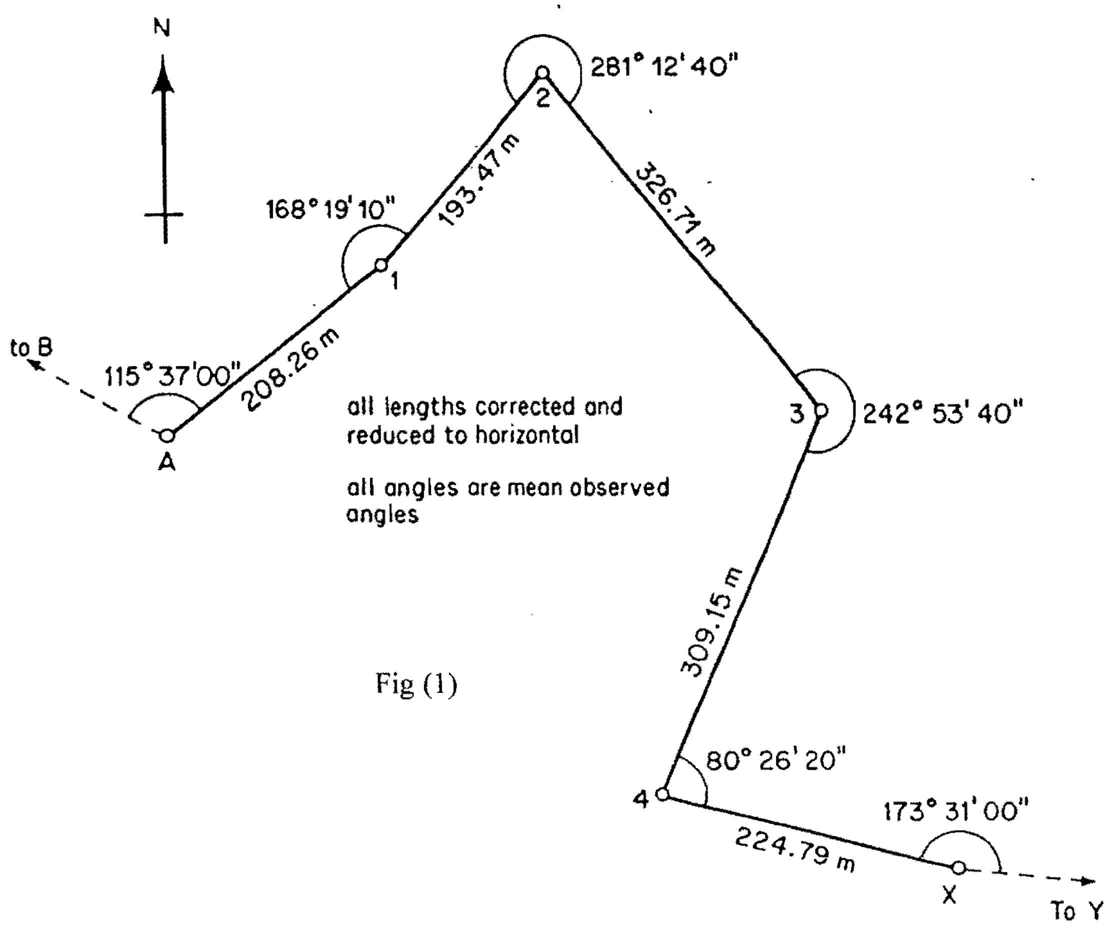

Fig (1)

2) Repeat Exercise (1) for fig(2)

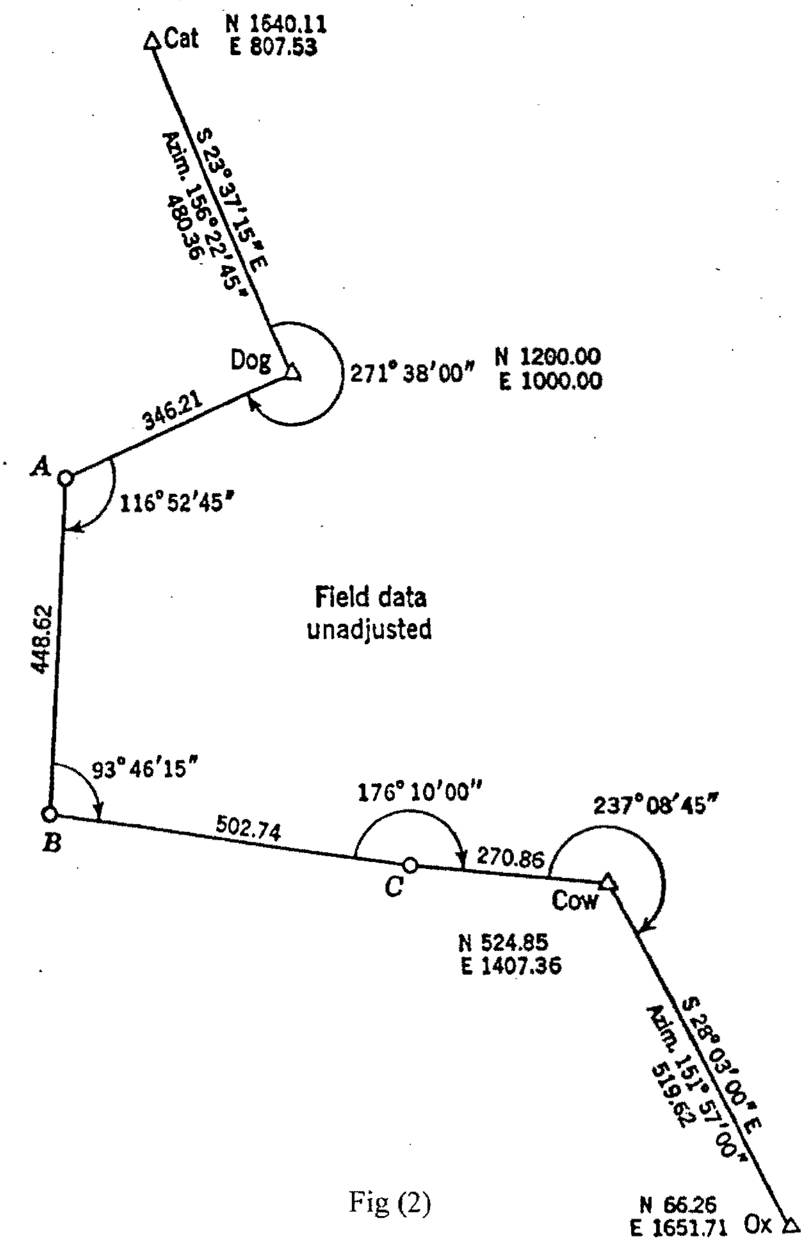

Fig (2)

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Engineering Surveying
Exercise No. 5

Simple Levelling

1- The following list of readings was taken in sequence during a levelling survey

Reading (m)	Remarks
1.250	BM, 1.435m AOD
1.285	Peg.A
1.125	Peg.B
0.810	Change point
1.555	
1.40	Inverted staff reading taken to the underside of a bridge
1.235	Peg.C
0.666	Change point
1.905	
0.070	BM, 4.600m AOD

Adopt a standard form of booking and reduce the levels to Ordnance datum.

2- The following table shows the readings taken to determine the clearance between the river level and the soffit of a road bridge. Reduce the levels and determine the clearance between the river level and the saffit of the bridge.

BS	IS	FS	RL	Remarks
0.872			21.460	OBM
0.665		3.980		
	2.920			River level at A
	-1.332			Soffit of bridge at A
	-1.312			Soffit of bridge at B
	-1.294			Soffit of bridge at C
	-1.280			Soffit of bridge at D
	2.920			River level at D
4.216		0.597		
		1.155		OBM

3- Complete the following missing data of a level book and carry out the normal check.

BS	IS	FS	Rise	Fall	Reduced Level
3.786					36.642
	1.312		2.474		39.116
	1.960			0.648	38.468
0.878		3.560		1.6-1	36.868
	3.698			2.826	34.042
	0.670		3.028		37.0-7
2.238		2.180		1.512	35.560
	1.052		1.186		36.746
2.874		2.808		1.754	34.992
	1.716		1.158		36.15
	0.950		0.766		36.916
		1.412		0.462	36.454

4- The following table shows the staff readings obtained from a survey along the line of proposed roadway from A to B.

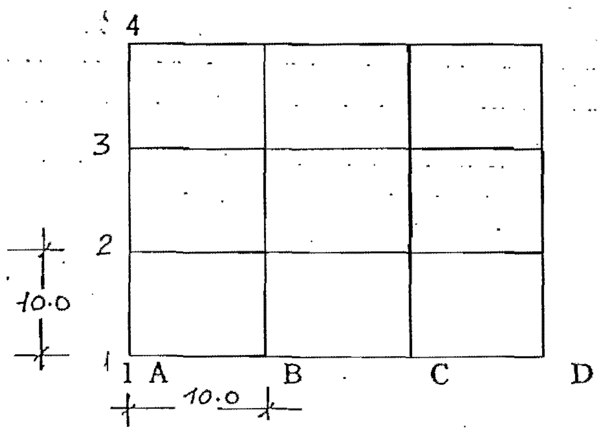
i) On the table, reduce the levels from A to B applying all the appropriate checks.

ii) Given that the finished rodway has to be evenly graded from A to B, calculate the depths of cut or fill at 20 meter intervals between A and B.

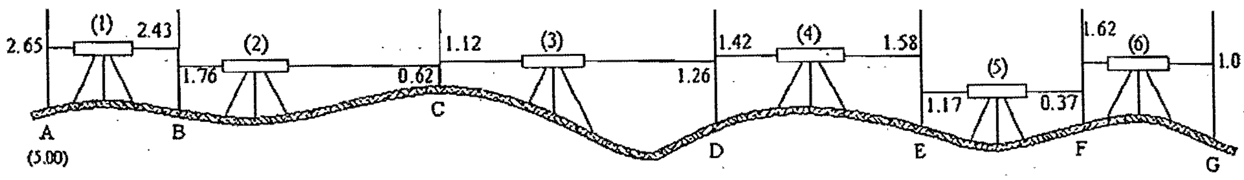
BS	*IS*	*FS*	*Rise Fall Or HPC*	*Surfacer reduced level*	*Grade Reduced Level*	*Fill*	*Cut*	*Remarks*
0.824				39.220				BM1
	1.628							A
	0.790							20 m from A
	0.383							40 m from A
2.154		1.224						60 m from A
	2.336							80 m from A
	2.757							100m from A
2.555		0.461						Change point
	2.275							120m from A
	0.436							140m from A
	0.227							160m from A
	0.716							180m from A
	0.652							B,200m from A
		0.233						BM2

5 - The following table shows the values of readings, taken on the leveling survey, of an area that is to be developed as part of a new project (the area shown in the figure), reduce the levels and apply check. Interpolate the ground contours, at 1 meter vertical intervals using scale 1:200 for the grid. Draw section 4D using appropriate scale.

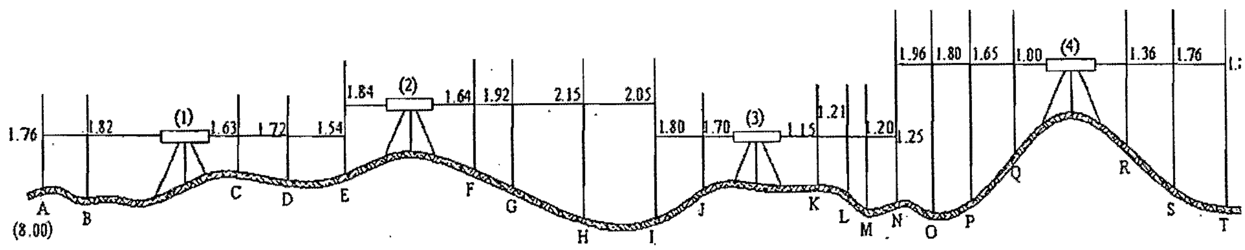
BS	*IS*	*FS*	*Rise Fall Or HPC*	*Reduced Level*	*Remarks*
1.320				26.34	Bench Mark
	0.290				A4
	0.910				B4
	2.170				C4
	3.620				D4
	1.920				A3
0.830		3.010			B3
	1.780				C3
	2.650				D3
	1.930				A2
	2.480				B2
0.610		3.240			C2
	1.570				D2
	1.840				A1
	1.840				B1
	1.850				C1
		3.320			D1



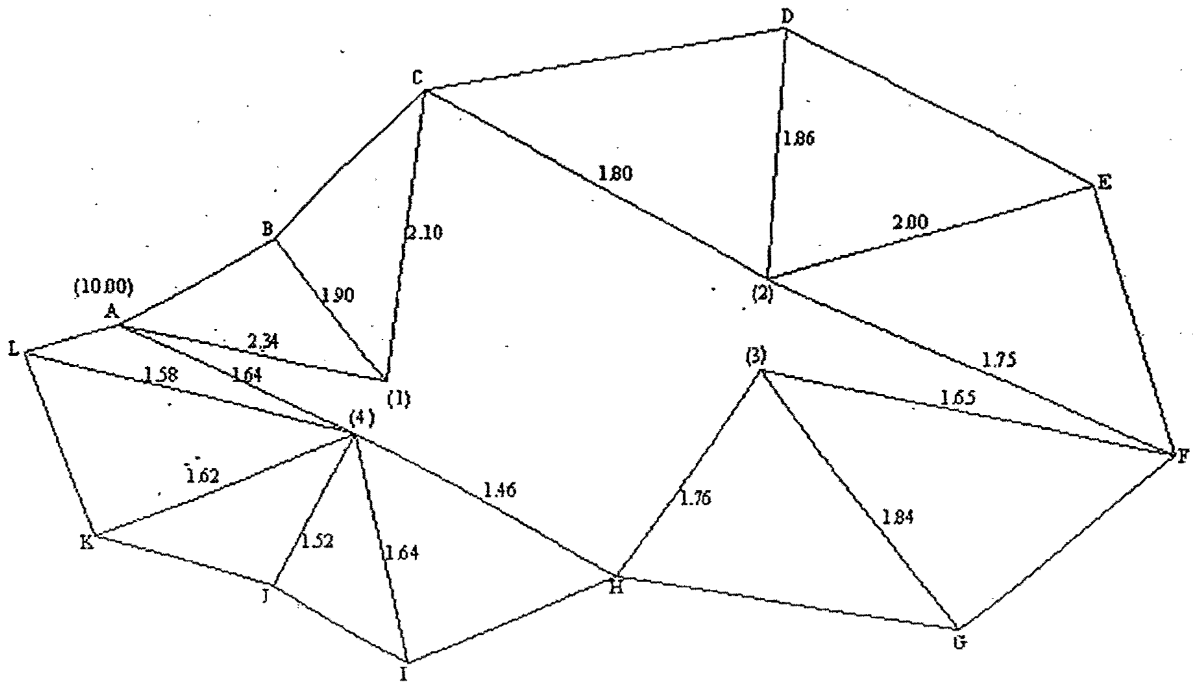
1- Figure (1) shows a direct application for level measurement between two points.
Use the method of rise & fall to determine the level for the seven points as shown.
[The mathematical check is required].



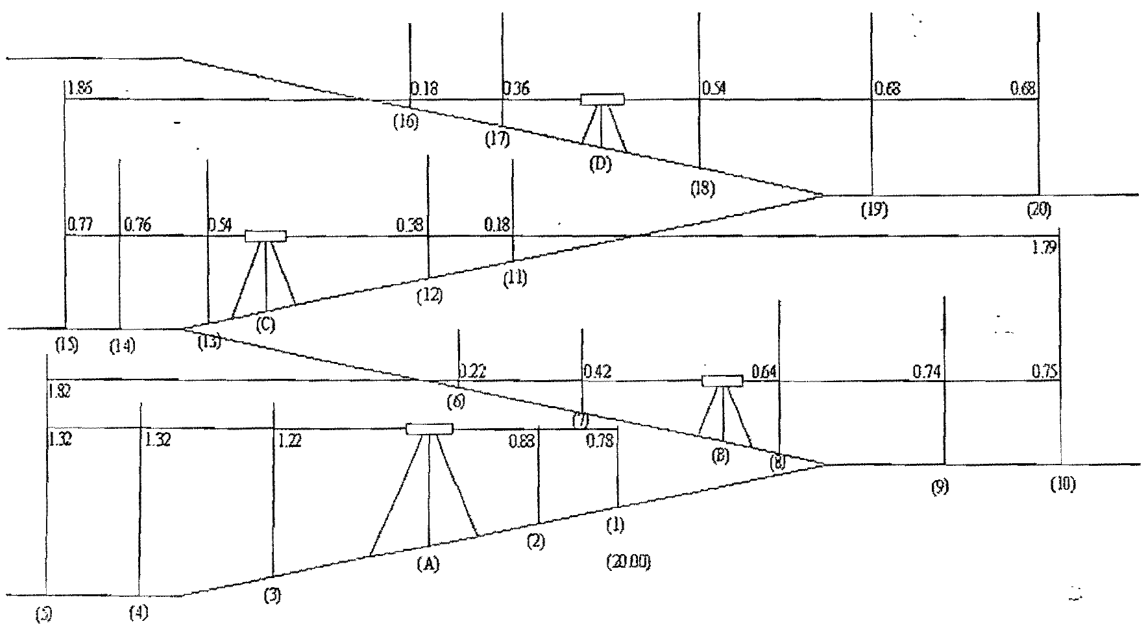
2- Figure (2) shows a similar level problem as in figure (1).
Use the HPC method to determine the level for the various points as shown.
[The mathematical check is required].

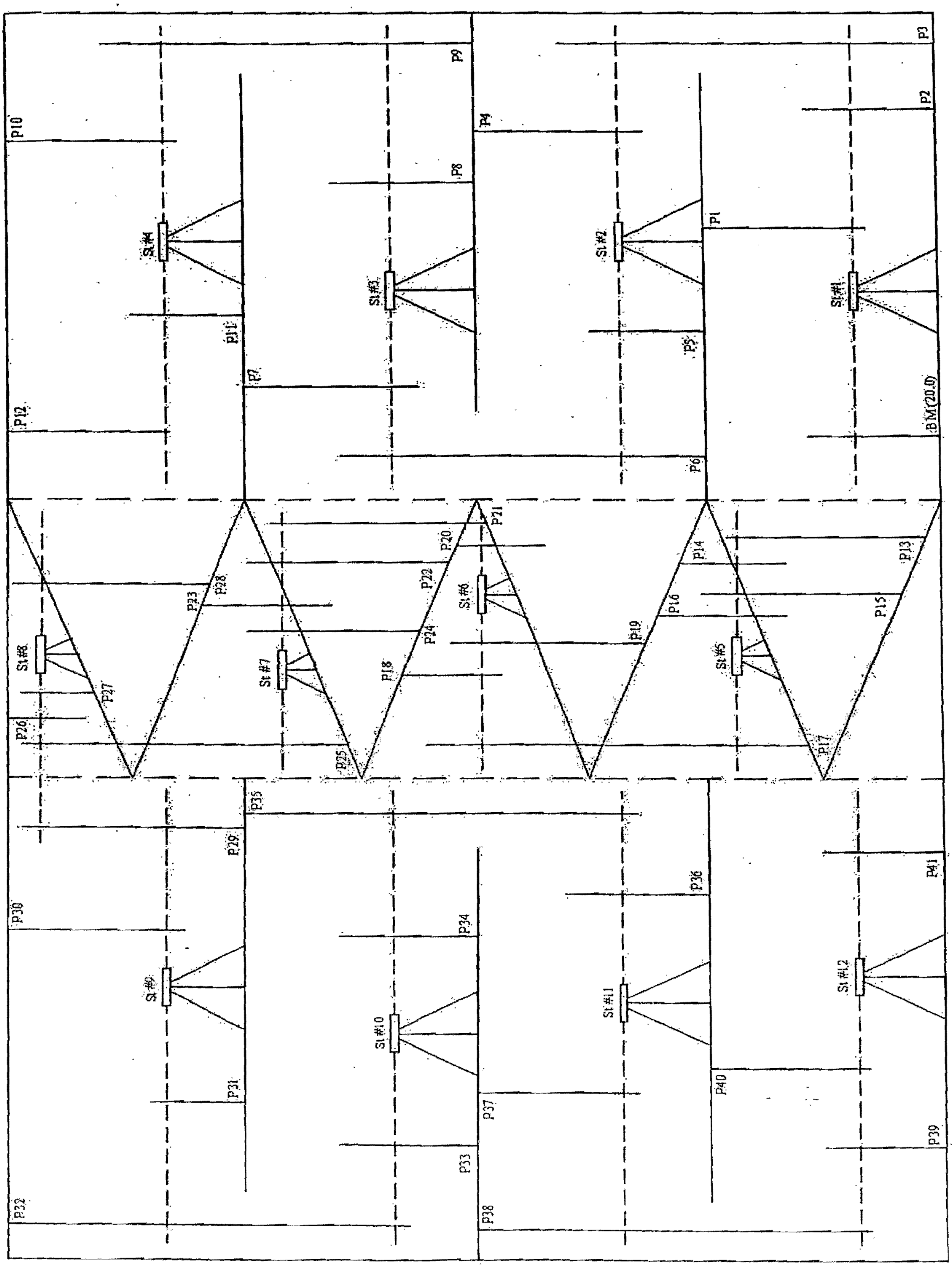


3- Figure (3) shows a closed traverse ABCDEFGHIJKLA. In order to determine the level of its various stations, the project engineer selects the level positions and conduct the level process as indicated in the figure. Use the data shown to determine the level of the traverse stations by using the rise & fall method.



4- Figure (4) shows a part of a construction project under development, internal stairs. It is required to check the levels of the various parts to compare with the consultant drawings. Use the data shown in the figure to determine the levels of the various points using the HPC method.




St #1
St #2
St #3
St #4
St #5
St #6
St #7
St #8
St #9
St #10
St #11
St #12
BM(20.0)
P1
P2
P3
P4
P5
P6
P7
P8
P9
P10
P11
P12
P13
P14
P15
P16
P17
P18
P19
P20
P21
P22
P23
P24
P25
P26
P27
P28
P29
P30
P31
P32
P33
P34
P35
P36
P37
P38
P39
P40
P41

Good Luck.

5- The Figure in page 4 shows a construction project, internal stairs.
It is required to check the levels of the various parts to compare with the consultant drawings.

- The reading of the point shows in the following table:

Point	Staff Reading	Point	Staff Reading
BM (20.0 m)	1.45	P21	0.07 / 2.98
P1	1.75	P22	2.09
P2	1.48	P23	1.21
P3	1.47 / 4.65	P24	1.65
P4	1.78	P25	0.83 / 4.32
P5	1.43	P26	0.27
P6	1.46 / 4.69	P27	1.24
P7	1.80	P28	2.45
P8	1.49	P29	3.10 /1.47
P9	1.46 / 4.68	P30	1.74
P10	1.79	P31	1.44
P11	1.46	P32	1.70 / 4.82
P12	1.82	P33	1.50
P13	2.95 (15 cm above BM)	P34	1.48
P14	1.62	P35	1.76 / 4.92
P15	1.98	P36	1.47
P16	2.12	P37	1.73
P17	0.63 / 4.78	P38	1.75 / 4.96
P18	1.34	P39	1.43
P19	2.07	P40	1.73
P20	0.32	P41	1.46

- Draw to scale the contour lines of the following leveling net if the vertical interval = 1:1000